§ 3.6 Note continued

Rational Inequalities

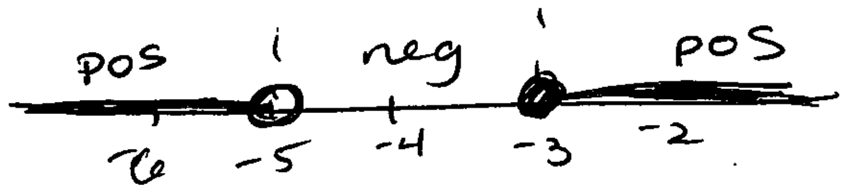
EXAMPLE Solve $\dfrac{x+3}{x+5} \geq 0$

SOLUTION

- zeros $\quad x+3=0$, $x=-3$
- V.A. $\quad x+5=0$, $x=-5$
- Sign chart

V.A. $x=-5$ $\qquad$ $x=-3$ zero

$y = \dfrac{x+3}{x+5}$



Test points

x	y
-6	$\dfrac{-6+3}{-6+5} = \dfrac{-3}{-1} = 3$ pos
-4	$\dfrac{-4+3}{-4+5} = \dfrac{-1}{1}$ neg
-2	$\dfrac{-2+3}{-2+5} = \dfrac{1}{3}$ pos

Answer

$(-\infty, -5) \cup [-3, \infty)$

or write

$\{x \mid x < -5 \text{ or } -3 \leq x\}$

EXAMPLE Solve: $\dfrac{1}{x+1} > \dfrac{2}{x-1}$

SOLUTION

$$\frac{1}{x+1} - \frac{2}{x-1} > 0$$

$$\frac{1}{(x+1)}\frac{(x-1)}{(x-1)} - \frac{2}{(x-1)}\frac{(x+1)}{(x+1)} > 0$$

$$\frac{(x-1) - 2(x+1)}{(x+1)(x-1)} > 0$$

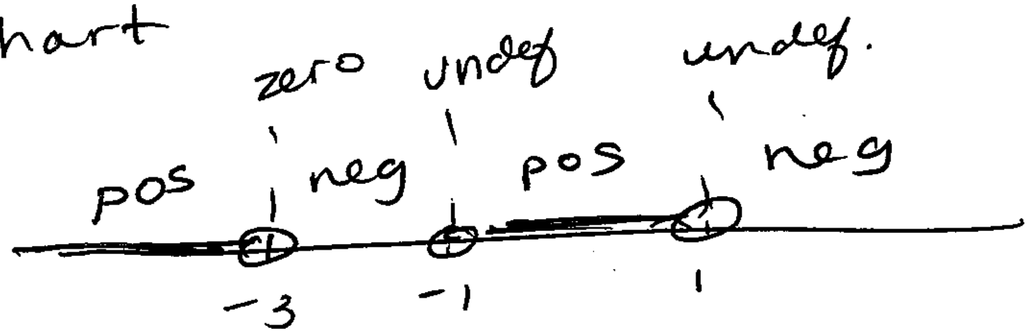
$$\frac{x - 1 - 2x - 2}{(x+1)(x-1)} > 0$$

$$\frac{-x-3}{(x+1)(x-1)} > 0$$

- Zeros $-x-3=0$
  $-x=3$
  $x=-3$

- Undef. (V.A) $x=-1,\ x=1$

- Sign Chart



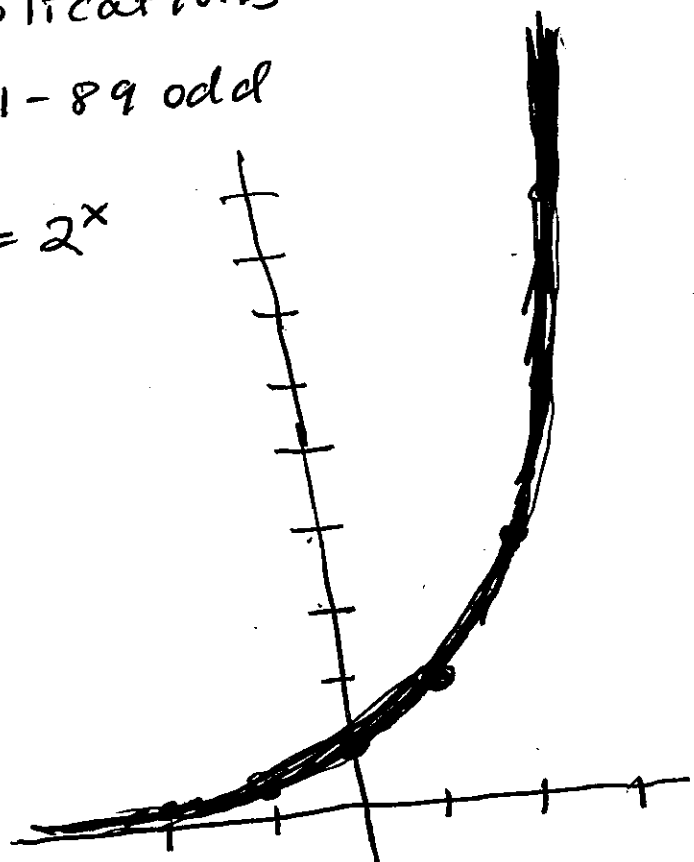
Test Pts

x	y
0	$\dfrac{(-0-3)}{(0+1)(0-1)} = \dfrac{-3}{-1} = 3$ pos

Answer: $(-\infty, -3) \cup (-1, 1$
or write
$\{x \mid x < -3$ or $-1 < x < 1$

§ 4.1 Exponential Functions and Their Applications

HW § 4.1 # 1-89 odd



EXAMPLE $f(x) = 2^x$

Plot Points

x	$y = 2^x$
-2	$2^{-2} = 1/2^2 = 1/4$
-1	$2^{-1} = 1/2$
0	$2^0 = 1$
$1/2$	$2^{1/2} = \sqrt{2}$
1	$2^1 = 2$
$3/2$	$2^{3/2} = (\sqrt{2})^3$
2	$2^2 = 4$
3	$2^3 = 8$
4	16
5	32
6	64
7	128
π	$2^{\pi} \approx 2^{3.14}$

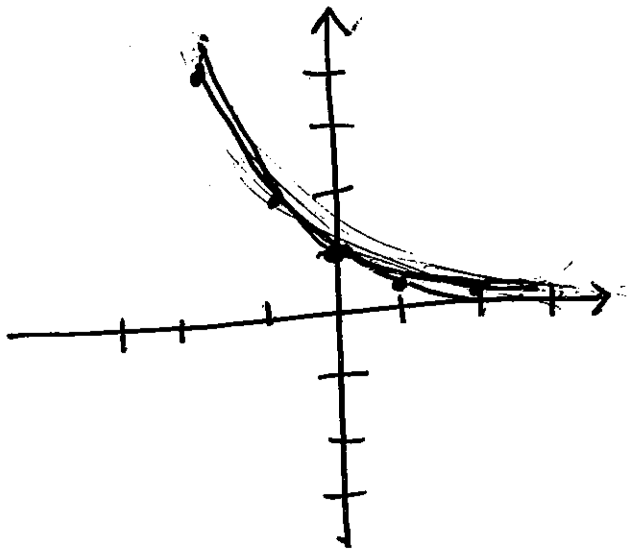
$y = 2^x$

- increasing
- one-to-one
- invertible
- domain: $\mathbb{R}$
- range: $y > 0$

EXAMPLE

$$y = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$$

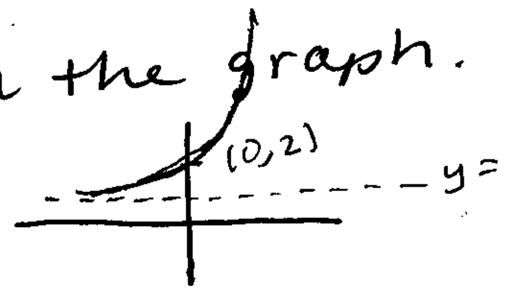
x	$y = 2^{-x}$
-2	$2^{-(-2)} = 4$
-1	$2^{-(-1)} = 2$
0	$2^{-0} = 1$
1	$2^{-1} = 1/2$
2	$2^{-2} = 1/4$



EXAMPLE Sketch the graph.

① $y = 2^x + 1$

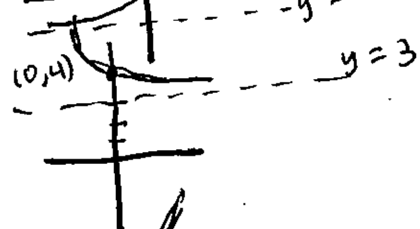
↑ shift up



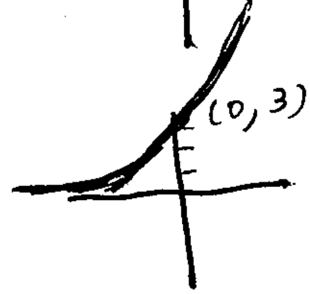
② $y = 2^x - 1$

↑ shift down

③ $y = 2^{-x} + 3$



④ $y = 3 \cdot 2^x$



x	$y = 2^x$
-2	$1/4$
-1	$1/2$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

x	$y = 3 \cdot 2^x$
-2	$3/4$
-1	$3/2$
0	3
1	6
2	4

EXAMPLE Solve.

① $4^{x} = \frac{1}{4}$

$$4^{(x)} = 4^{(-1)}$$

$$x = -1$$

② $8^{x-1} = \frac{1}{4^{x+2}}$

SOLUTION

$$(2^3)^{(x-1)} = \frac{1}{(2^2)^{(x+2)}}$$

$$2^{3(x-1)} = 2^{-2(x+2)}$$

$$3(x-1) = -2(x+2)$$

$$3x - 3 = -2x - 4$$

$$5x = -1$$

$$x = \frac{-1}{5}$$

Example: Let $f(x) = 5^{2-x}$.

Find x such that $f(x) = 125$.

SOLUTION

$125 = 5^3$

$$125 \to 25,\ 5 \qquad 25 \to 5,\ 5$$

$$f(x) = 125$$

$$5^{2-x} = 125$$

← write in terms of base 5.

$$5^{2-x} = 5^3$$

$$2 - x = 3$$

$$-x = 1$$

$$\boxed{x = -1}$$

Compound Interest

If P dollars are invested t years at an annual interest rate r compounded n times per year, then the final amount A is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

EXAMPLE You invest \$10,000 for 5 years at a rate of 3% compounded monthly. What is your ending balance?

SOLUTION

$P = 10{,}000$
$r = .03$
$n = 12$
$t = 5$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 10{,}000\left(1 + \frac{.03}{12}\right)^{12(5)}$$

$$\approx \$11{,}616.17$$

Explanation:

Month	0	1	2
A	10,000	$10{,}000 + 10{,}000\left(\frac{.03}{12}\right)$ $= 10{,}000\left(1 + \frac{.03}{12}\right)$	$10{,}000\left(1 + \frac{.03}{12}\right)^2$

3

$10{,}000\left(1 + \frac{.03}{12}\right)^3$ - - - -

$5 \cdot 12$ mos.

$10{,}000\left(1 + \frac{.03}{12}\right)^{5 \cdot 12}$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Next: Interest Compounded Continuously.

Let's look at the function

$y = \left(1 + \frac{1}{x}\right)^x$

Let's plot points

x	y
10	2.5937
100	2.7048
1000	2.7169
big number	≈ 2.718

$e \approx 2.71828183$

$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

$A = P(1+\frac{r}{n})^{nt}$

Let n get really big

As $n \to \infty$

$P\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t} \to Pe^{rt}$

Interest Compounded Continuously

$A = Pe^{rt}$

EXAMPLE You invest \$1000 for 40 years at a rate of 5% compounded continuously. What is the ending balance?

SOLUTION ~~$P = e^{rt}$~~ $A = Pe^{rt}$

$P = 1{,}000$

$r = .05$

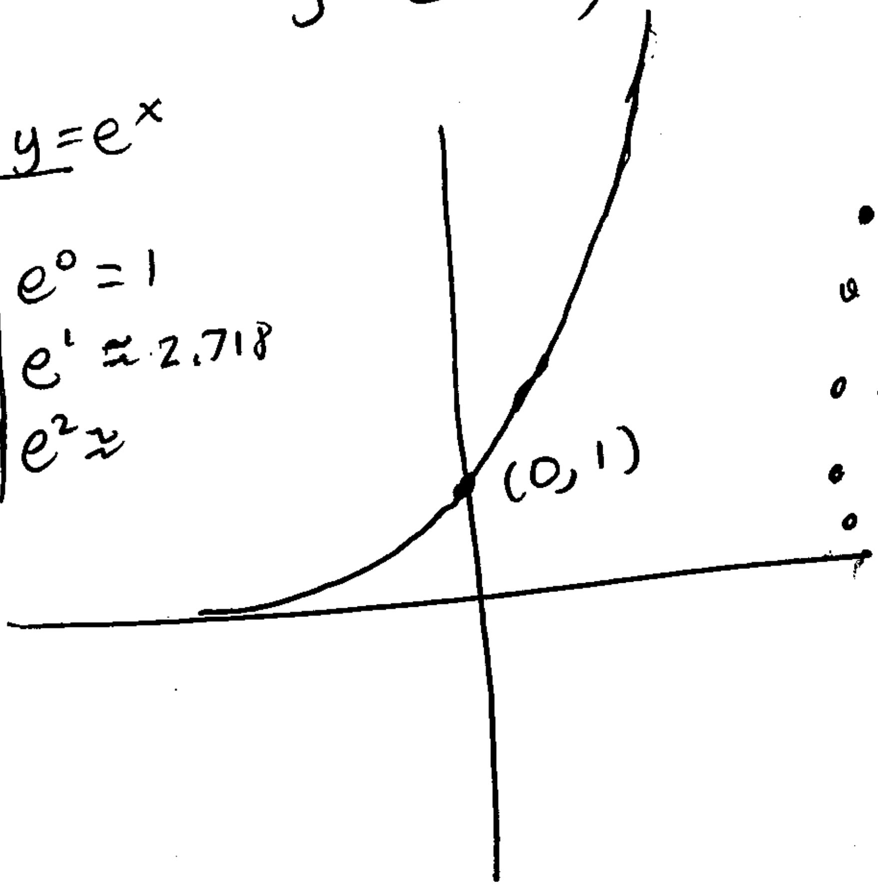
$t = 40$

$A = 1000\, e^{(.05)40} \approx 7{,}389.06$

Let's look at the function

$y = e^x$, $e \approx 2.718$

x	$y = e^x$
0	$e^0 = 1$
1	$e^1 \approx 2.718$
2	$e^2 \approx$



- Domain: $\mathbb{R}$
- Range: $y > 0$
- H.A.: The x-axis
- increasing
- invertible