

§3.5 Practice

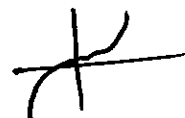
For the given polynomial

- State the zeros and give their multiplicity
- State the lead term
- Sketch the graph.

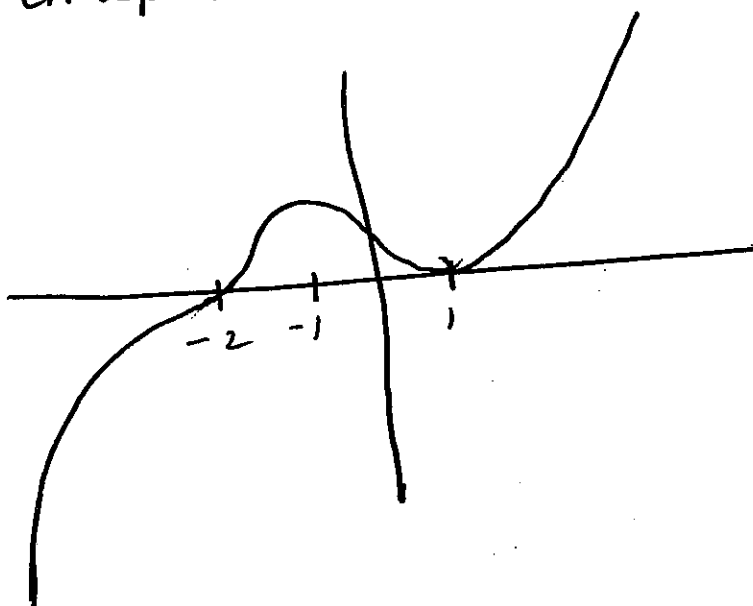
① $y = (x-1)^2(x+2)^3$

SOLUTION: zeros: $x=1$ m.2 touch
 $x=-2$ m.3 pass

Lead term $y = x^2 x^3 = x^5$

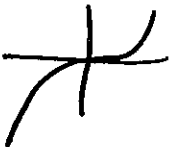


Graph

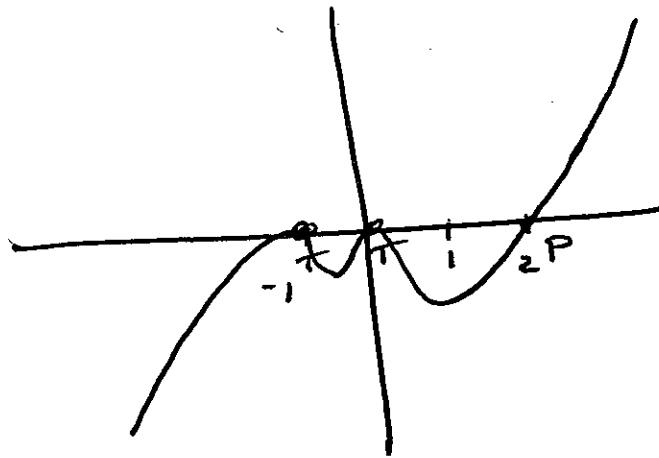


② $y = (x+1)^2 x^2 (x-2)$

SOLUTION: zeros: $x = -1$ m.2 touch
 $x = 0$ m.2 touch
 $x = 2$ m.1 pass

Lead term: $y = x^2 x^2 x = x^5$ 

Graph

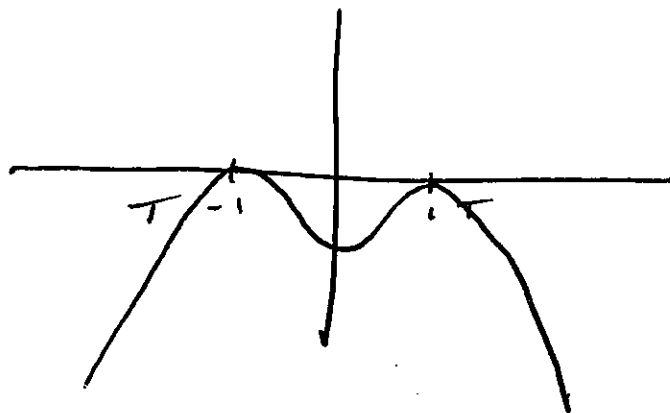
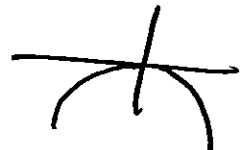


③ $y = -(x-1)^2 (x+1)^2$

SOLUTION: zeros: $x = 1$ m.2 touch
 $x = -1$ m.2 touch

Lead term $y = -x^2 x^2 = -x^4$

Graph



§3.6 For the given rational function

- a) Find the zeros
- b) Find the vertical asymptotes
- c) Find the horizontal or oblique asymptotes
- d) Make a sign chart
- e) Sketch the graph

④ $y = \frac{x-3}{x^2-1} =$

SOLUTION $y = \frac{x-3}{x^2-1} = \frac{x-3}{(x+1)(x-1)}$

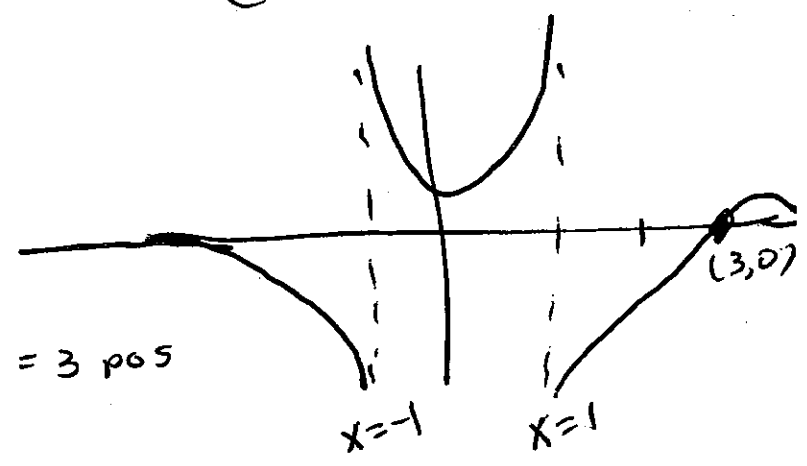
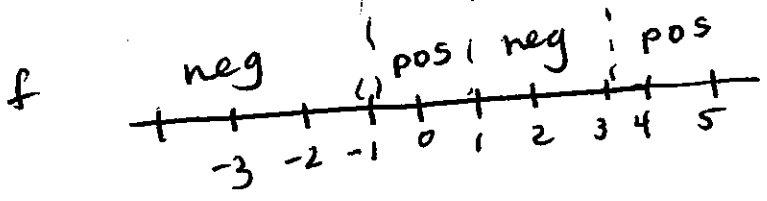
a) zeros: $x-3=0$
 $x=3$

b) Vertical asymptotes: $(x+1)(x-1)=0$
The lines $x=-1$ and $x=1$

c) Horiz Asymptotes
degree num < degree denom.
The line $y=0$ (x-axis)

e) VA $x=-1$ VA $x=1$ zero $x=3$

(f) Graph



Test points

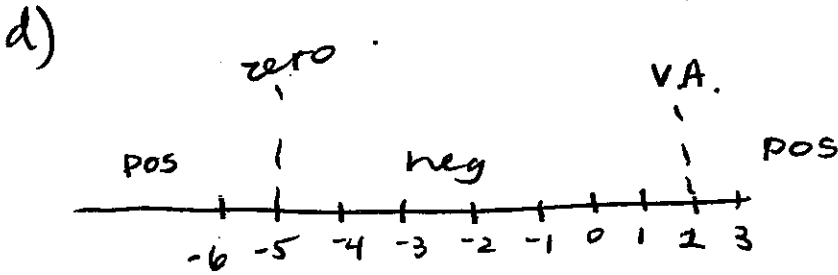
x	y
0	$\frac{-3}{0^2-1} = \frac{-3}{-1} = 3$ pos

⑤ $y = \frac{x+5}{x-2}$

a) zeros: $x+5=0$
 $x=-5$ x-int $(-5, 0)$

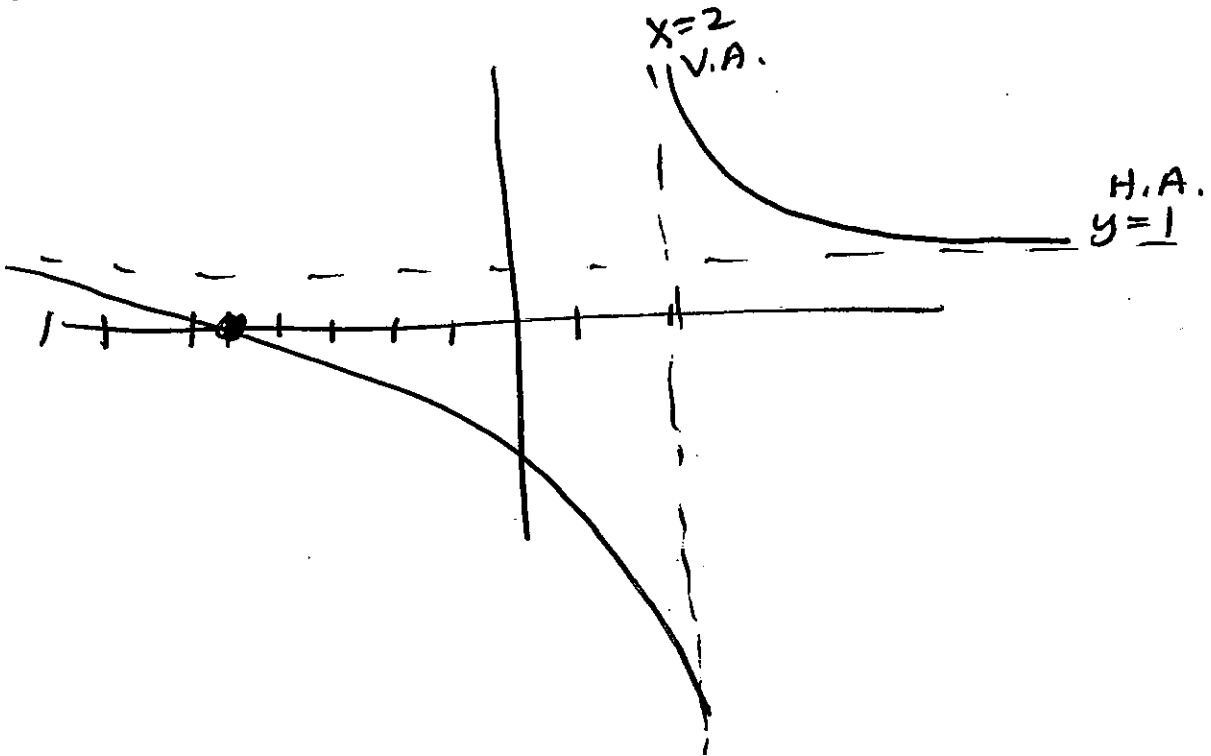
b) V.A. $x-2=0$
 The line $x=2$

c) H.A. deg num = deg denom
 $y = \frac{\text{lead coeff. num}}{\text{lead coeff. denom}} = \frac{1}{1} = 1$
 The line $y=1$



Test Pts	
x	y
-6	$\frac{-6+5}{-6-2} = \frac{-1}{-8}$ POS
0	$\frac{0+5}{0-2}$ neg
3	$\frac{3+5}{3-2}$ POS

e) Graph



⑥ $y = \frac{x^2 + x - 6}{x + 1}$

SOLUTION

a) x-int., zeros

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3, x = 2$$

$(-3, 0), (2, 0)$

b) V.A. $x + 1 = 0$
The line $x = -1$

c) oblique asymptote $\text{deg num} = \text{deg denom} + 1$

$$x + 1 \overline{) x^2 + x - 6}$$

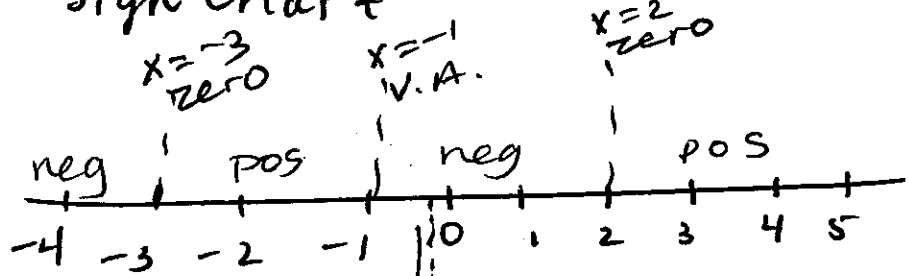
$$\underline{-x^2 + x} \quad \downarrow$$

$$0 - 6$$

$$y = x - \frac{6}{x + 1}$$

Oblig. Asympt. $y = x$ The line

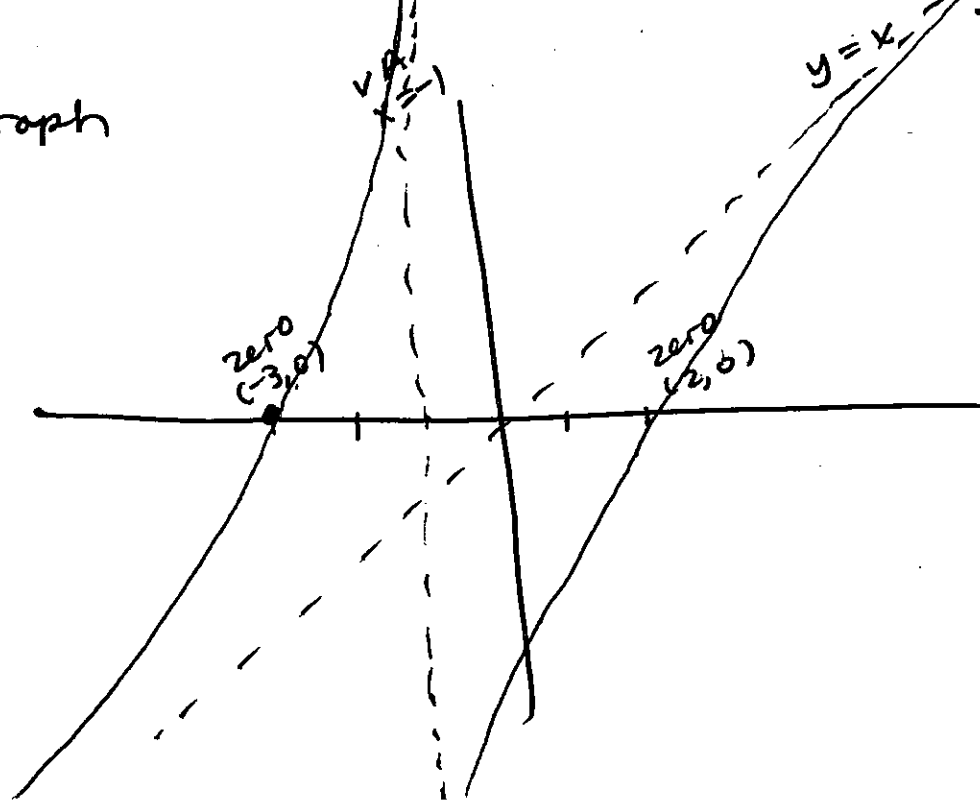
d) Sign Chart



Test pts

x	y
-4	$\frac{(-4+3)(-4-2)}{-4+1}$ $= \frac{6}{-3} = -2$
-2	4 pos
0	-6 neg
3	3/2 pos

e) Graph



§ 3.4 #17

$$\frac{1}{z} = \frac{3}{\sqrt{4z+1}}$$

SOLUTION

$$1 \cdot \sqrt{4z+1} = 3z$$

$$(\sqrt{4z+1})^2 = (3z)^2$$

$$4z+1 = 9z^2$$

$$9z^2 - 4z - 1 = 0$$

$$(\cancel{3z-1})(\cancel{z+1})$$

$$(\cancel{9z+1})(\cancel{z-1}) = 0$$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(9)(-1)}}{2(9)}$$

$$= \frac{4 \pm \sqrt{16+36}}{18}$$

$$= \frac{4 \pm \sqrt{52}}{18} = \frac{4 \pm \sqrt{4\sqrt{13}}}{18}$$

$$= \frac{4 \pm 2\sqrt{13}}{18}$$

$$= \frac{2(2 \pm \sqrt{13})}{2 \cdot 9}$$

$$= \frac{2 \pm \sqrt{13}}{9}$$

$$\begin{array}{l} 52 = 4 \cdot 13 \\ \wedge \\ 2 \quad 26 \\ \wedge \\ 2 \quad 13 \end{array}$$

For $\frac{1}{z} = \frac{3}{\sqrt{4z+1}}$

$$z \neq 0$$

and

$$4z+1 > 0$$

$$4z > -1$$

$$z > -\frac{1}{4}$$

SOL $z = \frac{2 \pm \sqrt{13}}{9}$

$$z = \frac{2 + \sqrt{13}}{9},$$

$$z = \frac{2 - \sqrt{13}}{9} \approx -0.178$$

check

$$\frac{1}{z} = \frac{3}{\sqrt{4z+1}}$$

$$\frac{1}{\left(\frac{2 - \sqrt{13}}{9}\right)} \stackrel{?}{=} \frac{3}{\sqrt{4\left(\frac{2 - \sqrt{13}}{9}\right) + 1}}$$

$$\text{neg } \left(\frac{9}{2 - \sqrt{13}}\right) = \frac{3}{\sqrt{\frac{8}{9} - \frac{4\sqrt{13}}{9} + 1}}$$

$$-5.6 \approx \text{pos } 1.75$$

Answer $z = \frac{2 + \sqrt{13}}{9}$