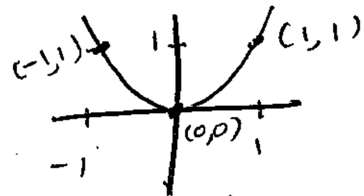
§ 3.5 Graphs of Polynomial Functions

HW § 3.5 # 1-91 odd.

Power Functions: $y = x^n$.

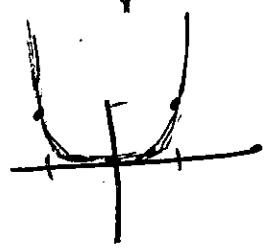
$y = x^n$, n is even.

$y = x^2$



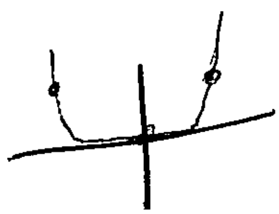
x	$y = x^2$
-1	1
0	0
1	1
½	¼
2	$2^2 = 4$

$y = x^4$



x	$y = x^4$
-1	1
0	0
1	1
½	$(\frac{1}{2})^4 = \frac{1}{16}$
2	$2^4 = 16$

$y = x^{10}$

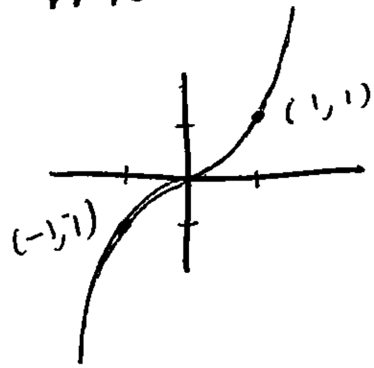


As $x \to \infty$, $y \to \infty$ i.e. $x^2 \to \infty$

As $x \to -\infty$, $y \to \infty$ i.e. $x^2 \to \infty$

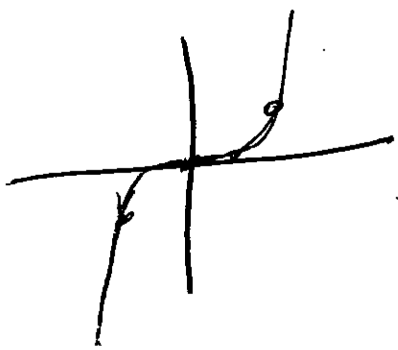
$y = x^n$, n is odd

$y = x^3$



x	$y = x^3$
-1	-1
0	0
1	1
$1/2$	$1/2^3 = 1/8$
2	$2^3 = 8$

$y = x^5$



x	$y = x^5$
-1	-1
0	0
1	1
$1/2$	$1/2^5 = 1/32$
2	$2^5 = 32$

As $x \underset{\text{approaches}}{\longrightarrow} \infty$, $y \to \infty$ that is $x^3 \to \infty$

. we write $\lim_{x \to \infty} x^3 = \infty$

As $x \to -\infty$, $y \to -\infty$ i.e. $x^3 \to -\infty$

FACT: For large values of x, polynomials behave like their highest power term.

EXAMPLE $y = x^3 - 10x^2 - 10x - 1$

As $x \to \infty$, y behaves like $y = x^3$

$y \to \infty$

Explanation

$$y = x^3 - 10x^2 - 10x - 1$$

$$= x^3\Big(1 - \underbrace{\frac{10}{x}}_{\downarrow\, 0} - \underbrace{\frac{10}{x^2}}_{\downarrow\, 0} - \underbrace{\frac{1}{x^3}}_{\downarrow\, 0}\Big) \quad \text{As } x \to \infty$$

$\approx x^3 \cdot 1$

when x is large.

Example: Let $y = x^4 - 10x^3 - 1$.

As $x \to \infty$, $y \to ?$

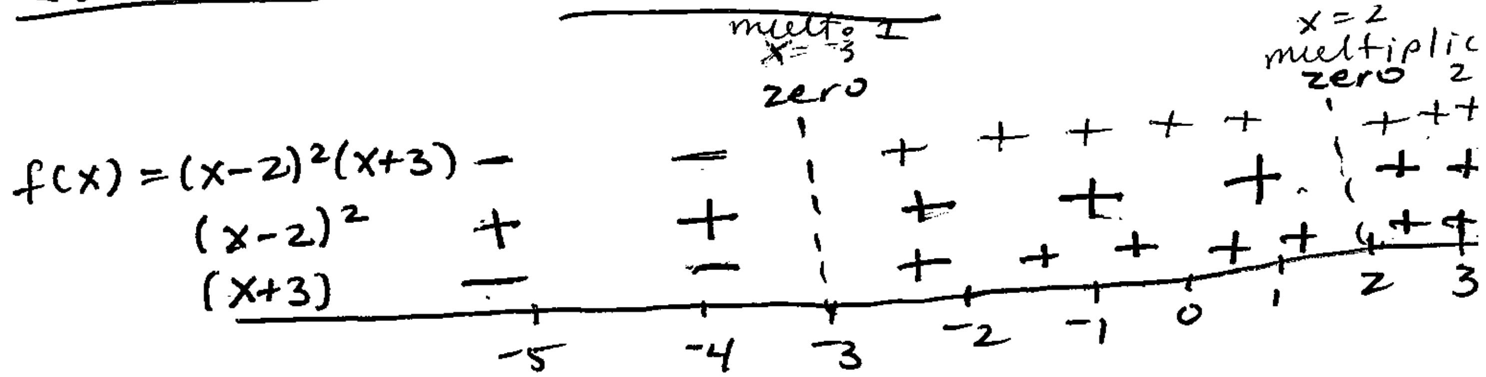
As $x \to \infty$, y behaves like $y = x^4$.

$x^4 \to \infty$

EXAMPLE Let $f(x) = (x-2)^2(x+3)$.

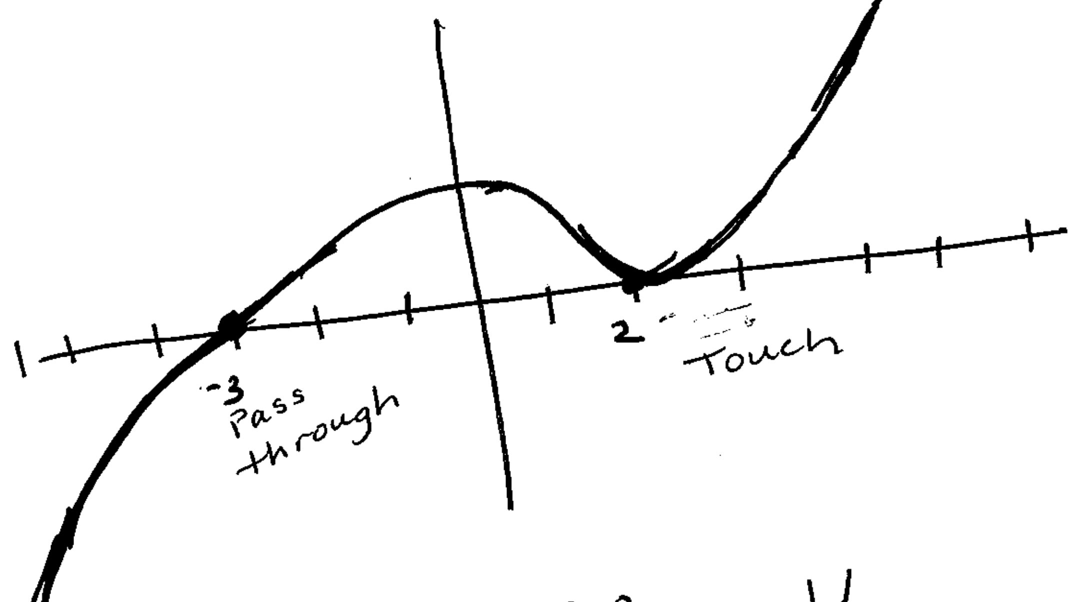
a) Make a sign chart showing the intervals where f is positive or negative.

SOLUTION Find the zeros: $x=2$, $x=-3$



b) Make a rough sketch.

zeros	multiplici
$x=2$	m. 2
$x=-3$	m. 1



c) Lead Term: $y = x^3$

- If the multiplicity of a zero is odd, then the graph passes through the x-axis at the zero
- If the multiplicity of a zero is even, then the graph touches the x-axis at the zero.

EXAMPLE Sketch the polynomial.

$$y = (x-1)^2(x+2)^3(x+5)$$

SOLUTION: zeros:

$x = 1$ m. 2 touch

$x = -2$ m. 3 pass

$x = -5$ m. 1 pass

Lead Term: $y = x^2\, x^3\, x$

$y = x^6$

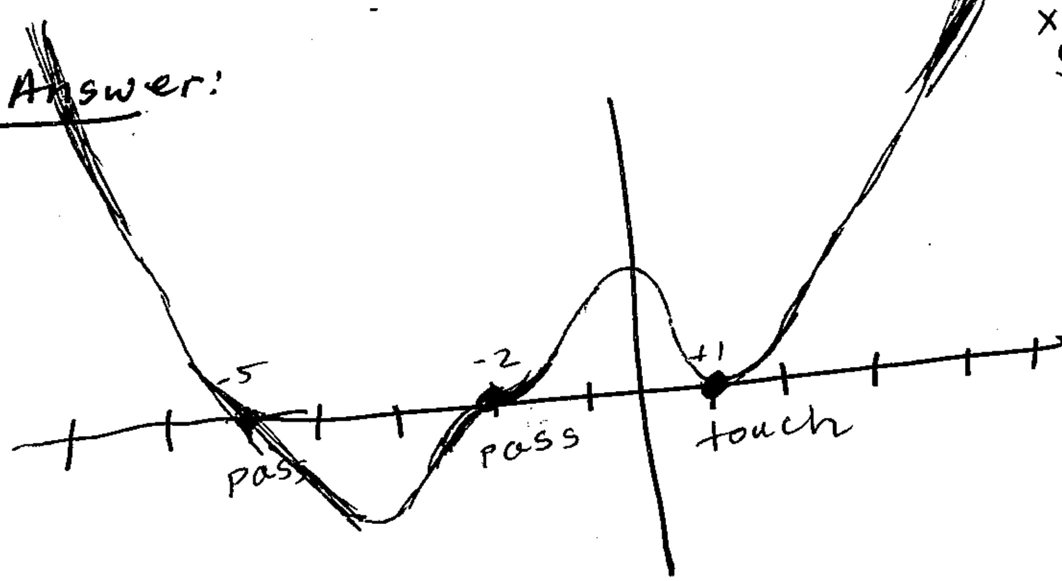
As $x \to \infty$

$y \to \infty$

Final Answer:

As $x \to -\infty$

$y \to \infty$



EXAMPLE Sketch the graph.

$$y = (x-3)^2(x+2)^3$$

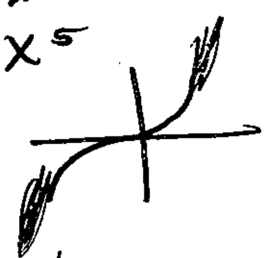
SOLUTION zeros

$x = 3$ mult. 2 touches

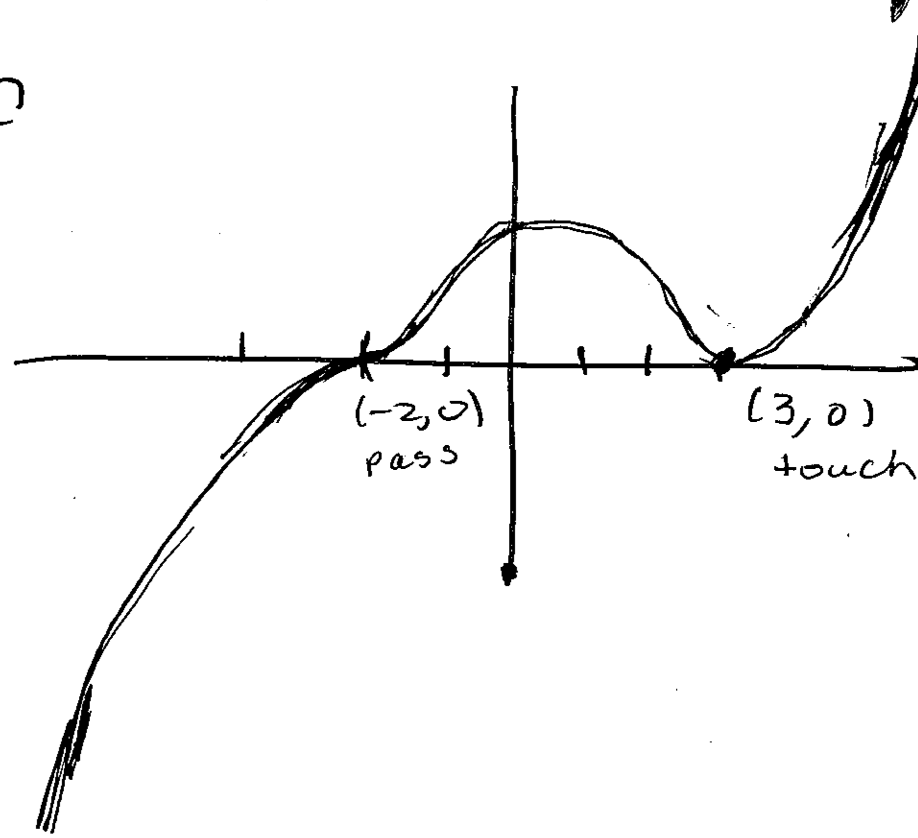
$x = -2$ mult. 3 pass

Lead Term $y = x^2 x^3$

$y = x^5$



Graph



§3.6 Rational Functions and Inequalities.

<u>EXAMPLE</u> Let $f(x) = \dfrac{x+1}{x-3}$

a) State the zero(s).

$$\frac{x+1}{x-3} = 0$$

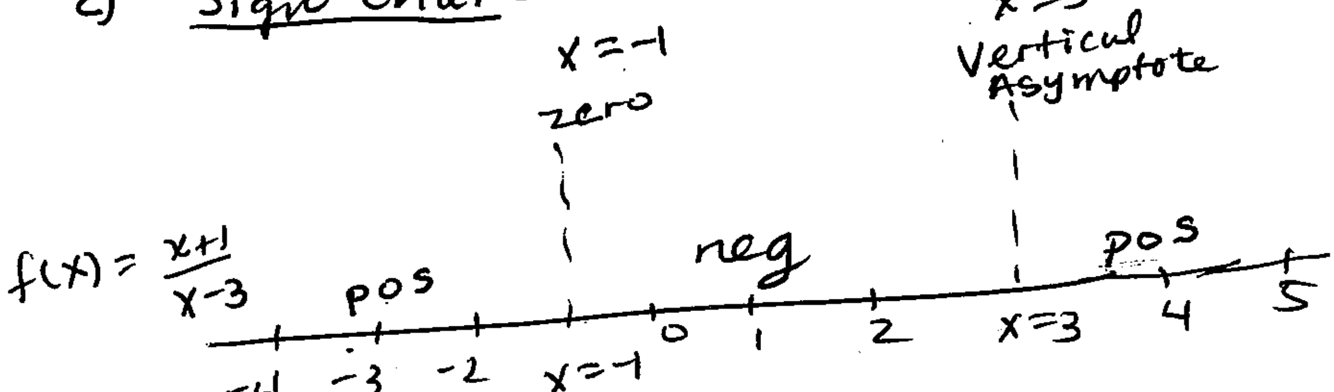
$$x+1=0$$
$$x=-1$$

b) Find the vertical asymptotes
(look for undefined numbers)

$y = \dfrac{x+1}{x-3}$ is undefined when $x-3=0$
$x=3$

Vertical Asymptote is the line $x=3$

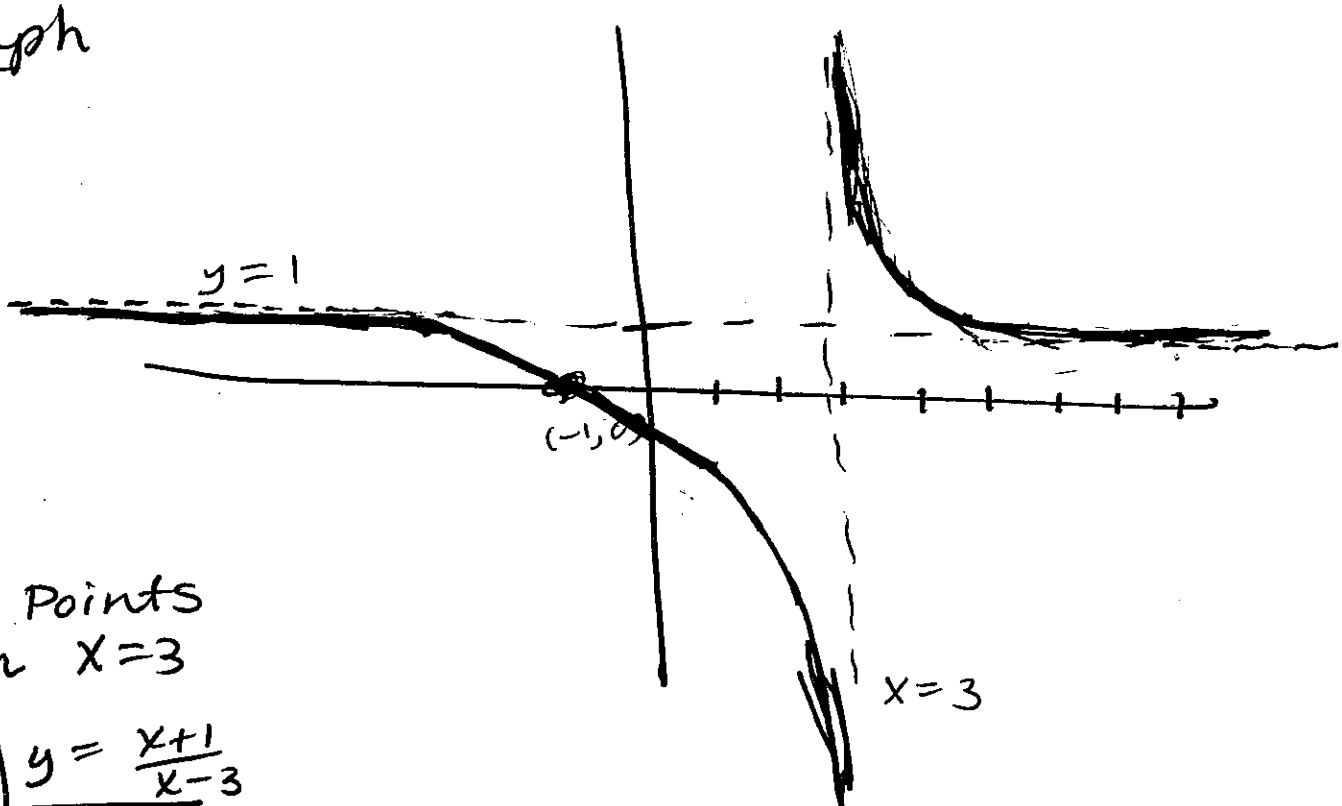
c) <u>Sign Chart</u>



<u>Test points</u>

x	y
0	$\frac{0+1}{0-3} = \frac{-1}{3}$ neg
-2	$\frac{-2+1}{-2-3} = \frac{-1}{-5} = 1/5$
4	$\frac{4+1}{4-3}$ pos

d) Graph



Plot Points near $x=3$

x	$y = \frac{x+1}{x-3}$
2.9	$2.9+1 / 2.9-3 = \frac{3.9}{-.1} = -39$
2.99	$\frac{2.99+1}{2.99-3} = \frac{3.99}{-.01} = -399$
3.01	$\frac{3.01+1}{3.01-3} = \frac{4.01}{.01} = 401$

$\frac{4}{\text{small}} = \text{big}$

e) Find the horizontal asymptotes if they exist.

$$y = \frac{x+1}{x-3}$$

$$y = \left(\frac{x+1}{x-3}\right)\frac{1/x}{1/x} = \frac{1 + 1/x}{1 - 3/x} \Rightarrow \frac{1}{1} = 1$$

as $x \to \infty$
$\frac{1}{x} \to 0$ and $3/x \to 0$

$y = 1$
horizontal asymptote.

- If the degree of the numerator equals the degree of the denominator then the horizontal asymptote is a fraction of the lead coefficients.

<u>Example</u>: $y = \frac{3x^2+2}{5x^2-1}$ Horiz Asympt. $y = \frac{3}{5}$

Back to horizontal asymptotes.

Example: $y = \dfrac{x+1}{x^2-3}$

Find the H.A.

SOLUTION: As $x \to \infty$ $\displaystyle\lim_{x\to\infty} \frac{x+1}{x^2-3} = 0$

$$y = \frac{x+1}{x^2-3} \to \frac{x}{x^2} = \frac{1}{x} \to 0$$

$\boxed{y=0}$

If the degree of numerator is less than the degree of the denominator then the horizontal asymptote is $y=0$ (x-axis)