§ 3.2 Practice.

Determine whether the given polynomial is a factor of the polynomial following it. If it is a factor, then factor the polynomial completely.

① $x+5$, $x^3+8x^2+11x-20$

SOLUTION

$$\begin{array}{r|cccc} -5 & 1 & 8 & 11 & -20 \\ & & -5 & -15 & 20 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$R=0$

yes, $x+5$ is a factor

$$x^3+8x^2+11x-20 = (x+5)(x^2+3x-4)$$

$$= (x+5)(x+4)(x-1)$$

□

Use the rational zero theorem to find all possible rational roots. Then find all real and imaginary zeros.

② $m(x) = x^3 + 4x^2 + 4x + 3$

SOLUTION Poss. Rat. Roots.

factors of 3: $\pm 1, \pm 3$

factors of 1: ± 1

Poss rat. roots: $\pm 1, \pm 3$

check

$$\begin{array}{r|cccc} 1 & 1 & 4 & 4 & 3 \\ & & 1 & 5 & 9 \\ \hline & 1 & 5 & 9 & 12 \end{array}$$ no

$$\begin{array}{r|cccc} -1 & 1 & 4 & 4 & 3 \\ & & -1 & -3 & -1 \\ \hline & 1 & 3 & 1 & 2 \end{array}$$ no

$$\begin{array}{r|cccc} -3 & 1 & 4 & 4 & 3 \\ & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$ yes

$= (x+3)(x^2 + x + 1)$

$x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$\left\{-3, \frac{-1 \pm i\sqrt{3}}{2}\right\}$$

③ $S(w) = w^4 + w^3 - w^2 + w - 2$

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SOLUTION Poss Rat Roots

$$= \frac{\text{Factors of } 2}{\text{Factors of } 1} = \pm 1, \pm 2$$

1	1	1	-1	1	-2
		1	2	1	2
	1	2	1	2	0

yes

$(x-1)(x^3+2x^2+x+2)$

x^3+2x^2+x+2 ← faster: factor by grouping

poss rat roots $\pm 1, \pm 2$

x^3+2x^2+x+2

$= x^2(x+2) + 1\cdot(x+2)$

$= (x+2)(x^2+1)$

$x = -2, \quad x^2+1=0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

1	1	2	1	2	
		1	3	4	
	1	3	4	6	NO

-1	1	2	1	2	
		-1	-1	0	
	1	1	0	2	NO

2	1	2	1	2	
		2	8	18	
	1	4	9	20	NO

-2	1	2	1	2	
		-2	0	-2	
	1	0	1	0	yes

$(x-1)(x+2)(x^2+1) = 0$

$x^2+1=0$

$x^2=-1$

$x = \pm\sqrt{-1} = \pm i$

$\{1, -2, \pm i\}$

§3.4 Find all real solutions to the equations.

① $\sqrt{x-1} = x-7$

SOLUTION $(\sqrt{x-1})^2 = (x-7)^2$

$x-1 = x^2-14x+49$

$x^2-15x+50=0$

$(x-10)(x-5)=0$

$x=10,\ x=5$

check: $x=10$ $\sqrt{10-1} \stackrel{?}{=} 10-7$

$\sqrt{9} \stackrel{?}{=} 3$ yes

check: $x=5$ $\sqrt{5-1} \stackrel{?}{=} 5-7$

$\sqrt{4} \stackrel{?}{=} -2$

$2 \stackrel{?}{=} -2$ NO

Answer: $x=10$

② $\sqrt{x+4} + \sqrt{x-1} = 5$

SOLUTION $\sqrt{x+4} = -\sqrt{x-1} + 5$

$(\sqrt{x+4})^2 = (-\sqrt{x-1} + 5)^2$

$x+4 = (x-1) - 10\sqrt{x-1} + 25$

$-20 = -10\sqrt{x-1}$

$(-20)^2 = (-10\sqrt{x-1})^2$

$400 = 100(\sqrt{x-1})^2$

$4 = x-1$

$\boxed{x=5}$

check $x=5$

$\sqrt{5+4} + \sqrt{5-1} \stackrel{?}{=} 5$

$\sqrt{9} + \sqrt{4} \stackrel{?}{=} 5$

$3+2 \stackrel{?}{=} 5$ yes

③ $x^{2/3} = \frac{1}{2}$

$(x^{2/3})^{3/2} = \pm(\frac{1}{2})^{3/2}$

$x = \pm\frac{1}{2^{3/2}}$

$x = \pm\frac{1}{2\sqrt{2}}$

$x = \pm\frac{1}{2}\frac{\sqrt{2}}{2} = \pm\frac{\sqrt{2}}{4}$

④ $w^{-3/2} = 27$

SOLUTION $(w^{-3/2})^{-2/3} = (27)^{-2/3}$

$$w = \left(\sqrt[3]{27}\right)^{-2}$$

$$w = 3^{-2}$$

$$w = \frac{1}{9}$$

⑤ $x^4 - 7x^2 + 10 = 0$

SOLUTION: $(x^2)^2 - 7(x^2) + 10 = 0$

let $u = x^2$, $u^2 - 7u + 10 = 0$

$(u-5)(u-2) = 0$

$u = 5, \quad u = 2$

$x^2 = 5, \quad x^2 = 2$

$x = \pm\sqrt{5}, \pm\sqrt{2}$

(6) $\left(\frac{b-5}{6}\right)^2 - \left(\frac{b-5}{6}\right) - 6 = 0$

SOLUTION Let $u = \frac{b-5}{6}$

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u = +3, \quad u = -2$$

$$\frac{b-5}{6} = +3, \quad \frac{b-5}{6} = -2$$

$$b - 5 = +18, \quad b - 5 = -12$$

$$b = 23, \quad b = -7$$