Equations Involving Square Roots

EXAMPLE Solve

$$\sqrt{x} + 2 = x$$

SOLUTION step 1
Isolate the radical

$$\sqrt{x} = x - 2$$

Step 2: Square both sides

$$(\sqrt{x})^2 = (x-2)^2$$
$$x = (x-2)^2$$
$$x = x^2 - 4x + 4$$

Step 3 Solve the quadratic.

$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1, \quad x = 4$$

Check: $\sqrt{x} + 2 = x$

$\cancel{x = 1}$ $\quad \sqrt{1} + 2 \stackrel{?}{=} 1$

$3 \neq 1$ NO

extraneous root.

$\underline{\underline{x = 4}}$ $\quad \sqrt{4} + 2 \stackrel{?}{=} 4$

$2 + 2 = 4$

$4 = 4$ yes

$$\boxed{\text{Answer } x = 4}$$

EXAMPLE Solve

$$\sqrt{2x+1} - \sqrt{x} = 1$$

SOLUTION

Step 1 Put one of the radicals on the other side.

$$\sqrt{2x+1} = \sqrt{x} + 1$$

Step 2 Square both sides

Aside
$(a+b)^2$
$= a^2 + 2ab + b^2$

$$(\sqrt{2x+1})^2 = (\sqrt{x} + 1)^2$$

$$2x + 1 = (\sqrt{x})^2 + 2(\sqrt{x})(1) + (1)^2$$

$$2x + 1 = x + 2\sqrt{x} + 1$$

Step 3 Isolate the radical

$$(2x+1) - x - 1 = 2\sqrt{x}$$

$$x = 2\sqrt{x}$$

Step 4 Square both sides

$$(x)^2 = (2\sqrt{x})^2$$

$$x^2 = 4x$$

Step 5 Solve

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0, \quad x = 4$$

<u>Check</u> $\sqrt{2x+1} - \sqrt{x} = 1$

<u>$x=0$</u> $\sqrt{2\cdot 0+1} - \sqrt{0} \stackrel{?}{=} 1$

$\sqrt{1} - 0 = 1$

$1 = 1$ yes

<u>$x=4$</u> $\sqrt{2\cdot 4+1} - \sqrt{4} \stackrel{?}{=} 1$

$\sqrt{9} - \sqrt{4} \stackrel{?}{=} 1$

$3-2 \stackrel{?}{=} 1$

$1 - 1$ yes

$\{0, 4\}$

<u>Equations with Rational Exponents</u>

<u>EXAMPLE</u> Solve.

① $x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

$625 = 5^4$

625

5 125

5 25

5 5

② $x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

③ $x^{4/3} = 625$

$(x^{4/3})^{3/4} = \pm(625)^{3/4}$

$x = \pm\left(\sqrt[4]{625}\right)^3$

$x = \pm\left(\sqrt[4]{5^4}\right)^3 = \pm 5^3 = \pm 125$

④ $(y-2)^{-5/2} = 32$

SOLUTION

$$\left((y-2)^{-5/2}\right)^{-2/5} = (32)^{-2/5}$$

$$y-2 = \frac{1}{32^{2/5}} = \frac{1}{(2^5)^{2/5}}$$

$$y-2 = \frac{1}{(\sqrt[5]{32})^2}$$

$$y-2 = \frac{1}{(2)^2} = \frac{1}{4}$$

$$y-2 = \frac{1}{4}$$

$$y = 2 + \frac{1}{4}$$

$$y = \frac{8}{4} + \frac{1}{4}$$

$$\boxed{y = \frac{9}{4}}$$

32

16 2

8 2

4 2

2 2

Equations of Quadratic Type

EXAMPLE Solve.

① $x^4 - 14x^2 + 45 = 0$

SOLUTION $(x^2)^2 - 14(x^2) + 45 = 0$

$u = x^2$

$u^2 - 14u + 45 = 0$

$(u-9)(u-5) = 0$

$u = 9, \quad u = 5$

$x^2 = 9, \quad x^2 = 5$

$x = \pm 3, \quad x = \pm\sqrt{5}$

$\{\pm 3, \pm\sqrt{5}\}$

② $x^4 - 9x^2 + 20 = 0$

SOLUTION $(x^2)^2 - 9(x^2) + 20 = 0$

Let $u = x^2$

$u^2 - 9u + 20 = 0$

$(u-5)(u-4) = 0$

$u = 5, \quad u = 4$

$x^2 = 5, \quad x^2 = 4$

$x = \pm\sqrt{5}, \quad x = \pm 2$

$\{\pm\sqrt{5}, \pm 2\}$

③ $x^{2/3} - 9x^{1/3} + 8 = 0$

SOLUTION

$$(x^{1/3})^2 - 9(x^{1/3}) + 8 = 0$$

$u = x^{1/3}$, $\quad u^2 - 9u + 8 = 0$

$$(u-8)(u-1) = 0$$

$$u = 8, \quad u = 1$$

$$x^{1/3} = 8, \quad x^{1/3} = 1$$

$$(x^{1/3})^3 = 8^3, \quad (x^{1/3})^3 = 1^3$$

$$x = 512, \quad x = 1 \quad \checkmark$$

Equations Involving Absolute Value

EXAMPLE Solve.

① $|x| = 2$

$$x = -2, \quad x = 2$$

② $2|x-3| = 8$

SOLUTION $\quad |x-3| = 4 \quad$ isolate abs value

$$x - 3 = \pm 4$$

$$x - 3 = 4 \quad \text{or} \quad x - 3 = -4$$

$$\boxed{x = 7 \quad \text{or} \quad x = -1}$$

③ $|x^2-6| = 5x$

SOLUTION

$x^2-6 = 5x$ or $x^2-6 = -5x$

$x^2-5x-6=0$ $\quad$ $x^2+5x-6=0$

$(x-6)(x+1)=0$ $\quad$ $(x+6)(x-1)=0$

$x=6,\ x=-1$ $\quad$ $x=-6,\ x=1$

Note: $5x$ must be nonnegative because it is equal to an absolute value.

$\{1, 6\}$