

Quadratic Inequalities

Example: Solve each inequality.

① $x^2 - x - 6 > 0$

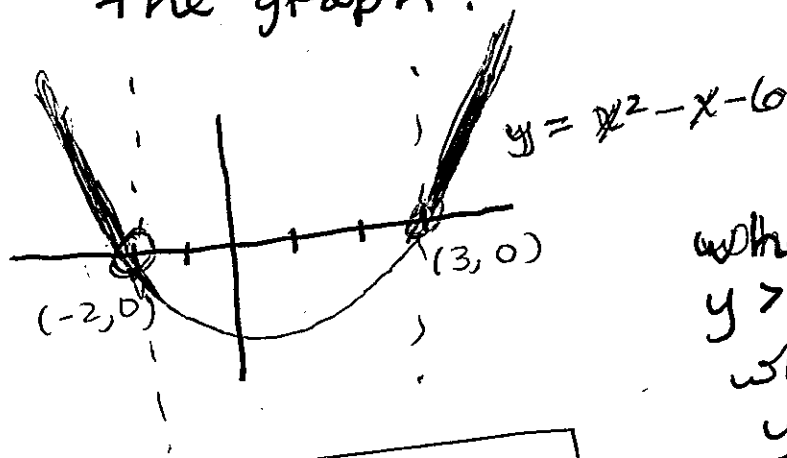
Step 1: Find x-int. of $y = x^2 - x - 6$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, x = -2$$

Step 2: Draw a rough sketch of the graph.



where is
 $y > 0$

where
 $y = x^2 - x - 6$

$$x < -2 \text{ or } 3 < x$$

$$(-\infty, -2) \cup (3, \infty)$$

② ~~1~~ ^{solve} $x^2 + x + 5 > 0$

Step 1: x-int.

$$x^2 + x + 5 = 0$$

$$b^2 - 4ac = 1^2 - 4(1)(5) \\ = -19$$

No real
sol.

No x-int.

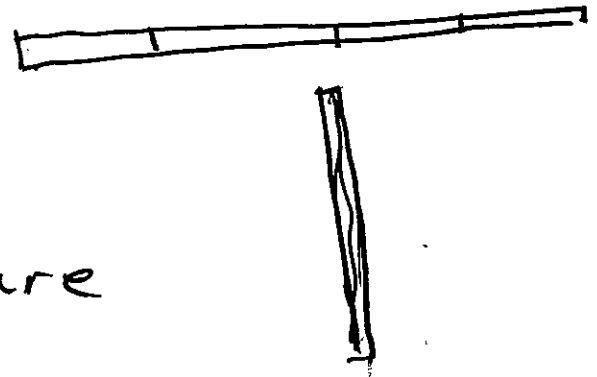
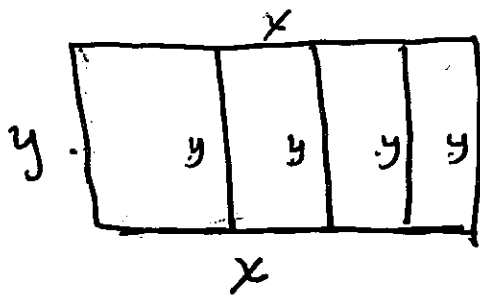
Step 2 Graph



SOLUTION: All real numbers,
 \mathbb{R}

EXAMPLE Kim wants to construct rectangular pens for four animals with 400 ft of fencing.

She will construct a rectangle pen with three interior fences parallel to one side. What overall dimensions will give the maximum area?



- SOLUTION
- Draw a picture
 - Formulas

Area: $A = xy$

$$2x + 5y = 400$$

- Eliminate a variable.

$$2x + 5y = 400$$

First solve $5y = -2x + 400$

for one of the variables, $y = \frac{-2}{5}x + \frac{400}{5}$

$$y = \frac{-2}{5}x + 80$$

$$A = xy$$

$$A = x\left(\frac{-2}{5}x + 80\right)$$

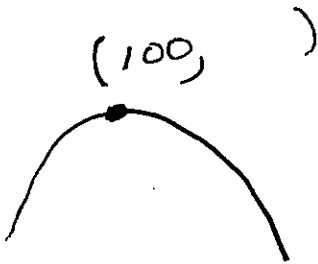
$$A = \frac{-2}{5}x^2 + 80x$$

- Find the ~~max~~ vertex

$$A = -\frac{2}{5}x^2 + 80x$$

$$h = \frac{-b}{2a} = \frac{-80}{2\left(-\frac{2}{5}\right)} = \frac{-80}{\left(-\frac{4}{5}\right)}$$

$$= -80 \cdot \frac{-5}{4} = 20.5 \cdot 4 = 100$$



$$\boxed{x = 100 \text{ ft}}$$

- Find y

$$x = 100$$

$$2x + 5y = 400$$

$$2(100) + 5y = 400$$

$$200 + 5y = 400$$

$$5y = 200$$

$$y = \frac{200}{5} = \cancel{80} 40$$

$$\boxed{y = 40}$$

$$\boxed{y = 40 \text{ ft}}$$

$$\boxed{40 \text{ ft by } 100 \text{ ft}}$$

Applications

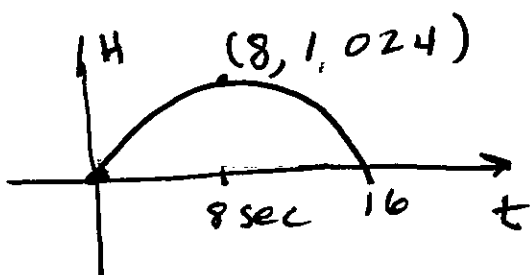
EXAMPLE If a soldier in basic training fires a rocket propelled grenade (RPG) straight up from ground level with initial velocity 256 ft/sec, then its height is given by

$$H(t) = -16t^2 + 256t$$

(a) What is the maximum height of the RPG?

SOLUTION Find the vertex.

$$h = -\frac{b}{2a} = \frac{-256}{2(-16)} = \frac{256}{32} = 8 \text{ sec.}$$



$$\begin{aligned} K = H(8) &= -16(8)^2 + 256(8) \\ &= -16(64) + 256(8) \\ &= 1,024 \text{ ft} \end{aligned}$$

Max height 1024 ft

b) How long does it take for the RPG to hit the ground?

SOL: Find t-int. $-16t^2 + 256t = 0$
 $-16t(t - 16) = 0$
 $t = 0, t = 16$

Answer: 16 sec

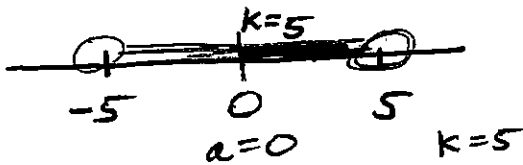
HW Review

§1.7

Write an inequality of the form $|x-a| < k$ or $|x-a| > k$ so that the inequality has the given solution set.

#71 $(-5, 5)$

$$-5 < x < 5$$



$$|x-0| < 5$$

Aside

$$|x-a| < k$$

$$-k < x-a < k$$

$$a-k < x < a+k$$

Algebraically

$$a-k = -5$$

$$a+k = 5$$

$$a = k - 5$$

$$a = -k + 5$$

$$k - 5 = -k + 5$$

$$2k = 10$$

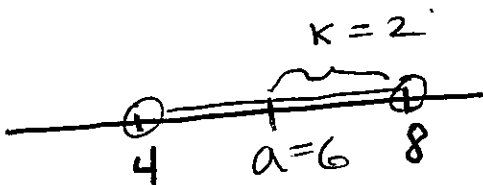
$$\boxed{k=5}$$

$$a - k = -5$$

$$a - 5 = -5$$

$$\boxed{a=0}$$

75 $(4, 8)$

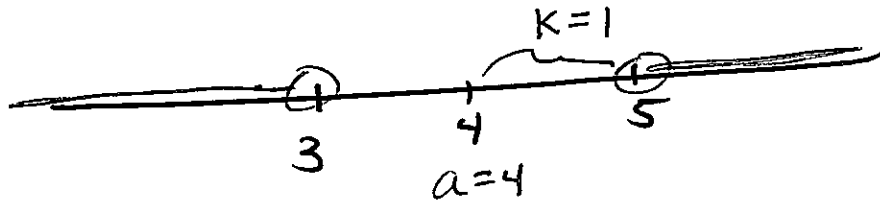


$$a = \frac{4+8}{2} = 6$$

$$|x-6| < 2$$

(77) $(-\infty, 3) \cup (5, \infty)$

$|x-a| > k$



$|x-4| > 1$