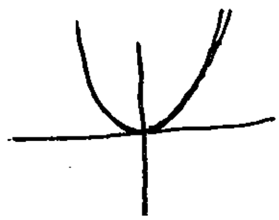
§3.1 Quadratic Functions and Inequalities; #1-87 odd.

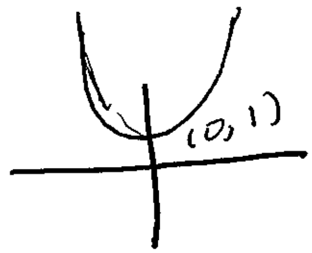
The Parabola

EXAMPLE: Sketch the graph

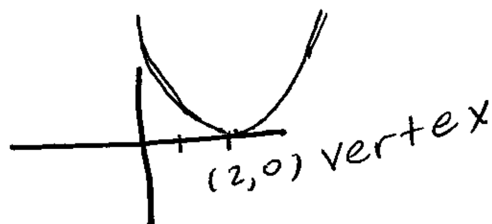
① $y = x^2$



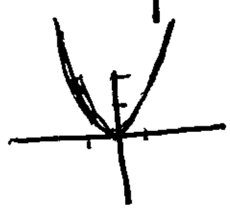
② $y = x^2 + 1$



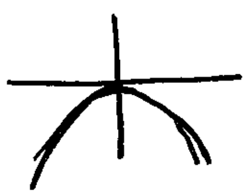
③ $y = (x-2)^2$



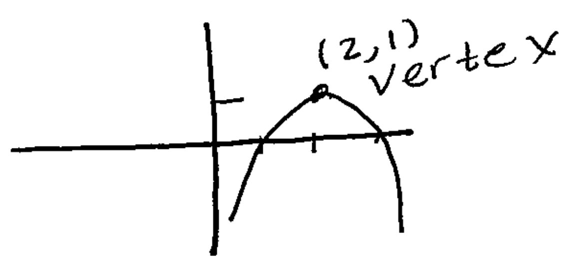
④ $y = 2x^2$



⑤ $y = -x^2$



⑥ $y = -(x-2)^2 + 1$



Standard Form of a Parabola.

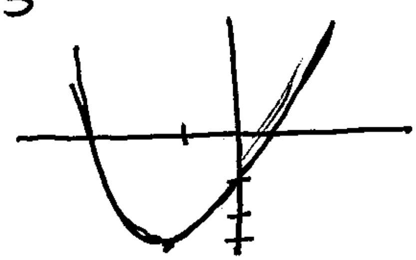
$$y = a(x-h)^2 + k$$

vertex (h, k)

EXAMPLE : Sketch, state the vertex.

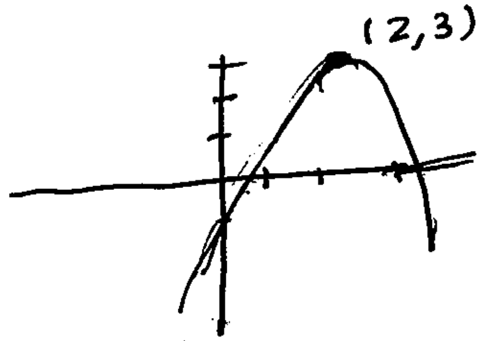
① $y = 2(x+1)^2 - 3$

sol vertex $(-1, -3)$



② $y = -(x-2)^2 + 3$ vertex: $(2, 3)$

sol



General Form for a Quadratic

$$y = ax^2 + bx + c$$

<u>EXAMPLE</u> Write the quadratic in the form $y = a(x-h)^2 + k$. State the vertex.

① $y = x^2 + 4x + 5$ We complete the square.

<u>SOL</u>

$$y = (x^2 + 4x + 4) - 4 + 5$$

$$4 = \left(\frac{4}{2}\right)^2$$

$$y = (x+2)^2 + 1$$

vertex $(-2, 1)$

<u>Aside</u>

$(x-y)^2 = x^2 - 2xy + y^2$

$(x-h)^2 = x^2 - 2hx + h^2$

② $y = x^2 + 3x + 1$

<u>SOL</u> $y = \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{1}{\frac{4}{4}}$

$$\frac{9}{4} = \left(\frac{3}{2}\right)^2$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$$

Vertex $\left(-3/2, -5/4\right)$

③ $y = -2x^2 + 16x + 1$

SOL $y = -2(x^2 - 8x + 16) + 32 \quad + 1$

$\left(\frac{-8}{2}\right)^2$

$y = -2(x-4)^2 + 33$

Vertex $(4, 33)$

Let's find a formula for h.

We start with

$$y = ax^2 + bx + c$$

$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{-b}{2a}\right)^2\right) - a\left(\frac{b}{2a}\right)^2 + c$$

$$\left(\frac{b}{a}\Big/{-2}\right)^2$$

$$h = \frac{b/a}{-2} = \frac{-b}{2a}$$

$$h = \frac{-b}{2a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 - a\frac{b^2}{4a^2} + c$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

$$\boxed{h = \frac{-b}{2a}}, \quad k = c - \frac{b^2}{4a}$$

Formula: $f(x) = ax^2 + bx + c$

then $h = \dfrac{-b}{2a}$ and $k = f(h)$

EXAMPLE Write in the form

$y = a(x-h)^2 + k$.

① $y = -2x^2 + 6x + 5$

SOL $\boxed{a = -2}$ $b = 6$, $c = 5$

$h = \frac{-b}{2a} = \frac{-6}{2(-2)} = \frac{6}{4} = \frac{3}{2}$ $\boxed{h = \frac{3}{2}}$

$k = f(h)$

$K = -2\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) + 5$

$= -2\left(\frac{9}{4}\right) + 3 \cdot 3 + 5$

$= -\frac{9}{2} + 14 = \frac{-9}{2} + \frac{28}{2} = \frac{19}{2}$

$\boxed{k = \frac{19}{2}}$

$y = a(x-h)^2 + k$

$\boxed{y = -2\left(x - \frac{3}{2}\right)^2 + \frac{19}{2}}$

§3.1 Notes continued.

x-intercepts

EXAMPLE Find the x-intercepts.

① $y = x^2 + 5x + 6$

SOLUTION: SOLVE

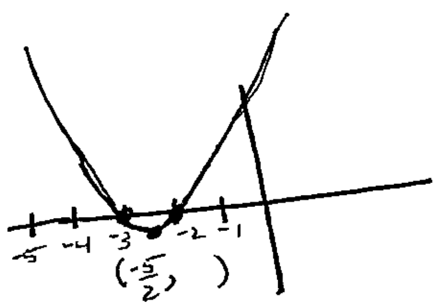
$$x^2 + 5x + 6 = 0$$

Factor: $(x+2)(x+3) = 0$

$x+2=0, \quad x+3=0$

$x=-2, \quad x=-3$

$$\boxed{(-2,0),\ (-3,0)}$$



h is the average of the x-intercepts.

$$h = \frac{-2 + -3}{2} = -5/2$$

② $y = x^2 - x - 6$

Find the x-int.

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, \quad x = -2$$

$$\boxed{(-2,0),\ (3,0)}$$

The Quadratic Formula

$ax^2 + bx + c = 0$ has solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof:

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + a\frac{b^2}{4a^2}$$

$$\left(\frac{b}{2a}\right)^2$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{1}{a}\left(\frac{b^2}{4a} - c\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b^2-4ac is called the discriminant

- if $b^2-4ac>0$ then there are two real solutions
- if $b^2-4ac=0$, then there is one real solution.
- if $b^2-4ac<0$, then there are zero real solutions (there are 2 imaginary solutions)

Example: Find the x-intercepts of the quadratic.

a) $y = x^2 + 5x^2 + 2$

SOLUTION: $a=1,\ b=5,\ c=2$

$$x = \frac{-(5) \pm \sqrt{(5)^2-4(1)(2)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25-8}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

b) $y = x^2 + x + 4$

Sol $a=1,\ b=1,\ c=4$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-16}}{2} = \frac{-1 \pm \sqrt{-15}}{2}$$

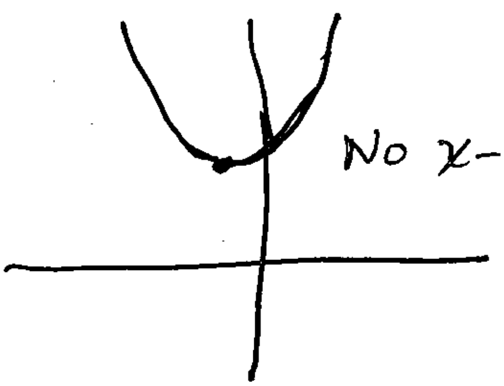
- No real sol.

Follow-up; $h = \frac{-b}{2a} = \frac{-1}{2(1)} = -\frac{1}{2}$

$$K = f(-\tfrac{1}{2}) = (-\tfrac{1}{2})^2 + (-\tfrac{1}{2}) + 4$$

$$= \frac{1}{4} - \frac{1}{2} + 4$$

$$= \frac{1}{4} - \frac{2}{4} + \frac{16}{4} = \frac{13}{4}$$



No x-intercepts.

Aside: • The imaginary number i is defined as $i = \sqrt{-1}$.

- A complex number is of the form $a+bi$, where a and b are real.

- An imaginary number is of the form $a+bi$, a, b real, $b \neq 0$.