Weds. 23 June 2010

§2.3 Practice.

Determine whether the function is odd, even, or neither.

1. $f(x) = x^4 - 3x^2$

<u>Sol</u> $f(-x) = (-x)^4 - 3(-x)^2$
$= x^4 - 3x^2$
$= f(x)$ even.

2. $f(x) = x^3 + 2x$

<u>Sol</u> $f(-x) = (-x)^3 + 2(-x)$
$= -x^3 - 2x$
$= -(x^3 + 2x)$
$= -f(x)$ odd

3. $f(x) = \dfrac{x^2-1}{x^2+1}$

<u>Sol</u> $f(-x) = \dfrac{(-x)^2-1}{(-x)^2+1} = \dfrac{x^2-1}{x^2+1} = f(x)$ even.

4. $f(x) = x^4 + 2x^3 + 1$

<u>Sol</u> $f(-x) = (-x)^4 + 2(-x)^3 + 1$
$= x^4 - 2x^3 + 1$
$\neq f(x)$
$\neq -f(x)$ neither.

§2.4 Practice

Let $f(x) = |x|$, $g(x) = x - 7$, $h(x) = x^2$. Write each of the following functions as a composition of functions chosen from f, g, and h.

5. $Q(x) = |x| - 7$

SOL $Q(x) = f(x) - 7$

$= g(f(x))$

$Q = g \circ f$

6. $M(x) = |x - 7|$

SOL $M(x) = |g(x)|$

$M(x) = f(g(x))$

$M = f \circ g$

7. $Q(x) = (x^2 - 7)^2$

SOL $Q(x) = (h(x) - 7)^2$

$= (g(h(x)))^2$

$Q(x) = h(g(h(x)))$

$Q = h \circ g \circ h$

8. $T(x) = x^4$

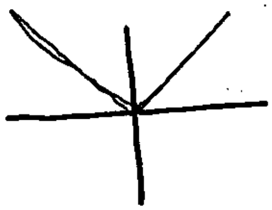
SOL $T(x) = (x^2)^2$

$T(x) = (h(x))^2 = h(h(x))$

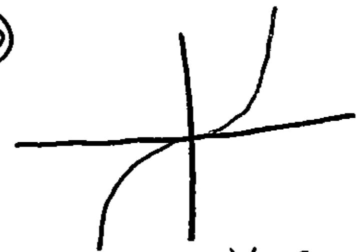
$T = h \circ h$

§2.5 Practice

Use the horizontal line test to determine whether the function is one-to-one.

(9) 

No

(10) 

Yes

Determine whether the function is one-to-one.

Example: $f(x) = 2x+3$

Sol Let $f(a) = f(b)$

$2a+3 = 2b+3$

$2a = 2b$

$a = b$ ✓ yes

(11) $h(x) = 4x - 9$

Sol Let $h(a) = h(b)$

$4a-9 = 4b-9$

$4a = 4b$

$a = b$ ✓ yes

(12) $g(x) = \dfrac{x+2}{x+3}$

Sol $g(a) = g(b)$

$\dfrac{a+2}{a+3} = \dfrac{b+2}{b+3}$

$(a+2)(b+3) = (a+3)(b+2)$

$ab + 3a + 2b + 6 = ab + 2a + 3b + 6$

$3a + 2b = 2a + 3b$

$a = b$ ✓ yes

§2.4 Find the inverse function by using the switch and solve method.

(13) $f(x) = -2x+5$

Sol $y = -2x+5$

$x = -2y+5$

$2y = -x+5$

$y = \frac{-x+5}{2}$

$f^{-1}(x) = -\frac{x}{2} + \frac{5}{2}$

(14) $f(x) = \sqrt{3x-1}$

Sol

$y = \sqrt{3x-1}\,,\quad y \geq 0$

$x = \sqrt{3y-1}\,,\quad x \geq 0$

$x^2 = 3y-1$

$x^2+1 = 3y$

$y = \frac{x^2+1}{3}$

$y = \frac{x^2}{3} + \frac{1}{3}\,,\quad x \geq 0$

$f^{-1}(x) = \frac{x^2}{3} + \frac{1}{3}\,,\quad x \geq 0$

(15) $f(x) = \dfrac{2x-1}{x-6}$

SOL

$$y = \frac{2x-1}{x-6}$$

$$x = \frac{2y-1}{y-6}$$

$$x(y-6) = 2y-1$$

$$xy - 6x = 2y - 1$$

$$xy - 2y = 6x - 1$$

$$y(x-2) = 6x-1$$

$$y = \frac{6x-1}{x-2}$$

$$f^{-1}(x) = \frac{6x-1}{x-2} \quad \square$$

Find $f(g(x))$ and $g(f(x))$. Then determine whether f and g are inverses.

(16) $f(x) = \frac{1}{x} + 3, \quad g(x) = \frac{1}{x-3}$

SOL • $f(g(x)) = \dfrac{1}{g(x)} + 3$

$$= \frac{1}{\left(\frac{1}{x-3}\right)} + 3 = x-3+3 = x$$

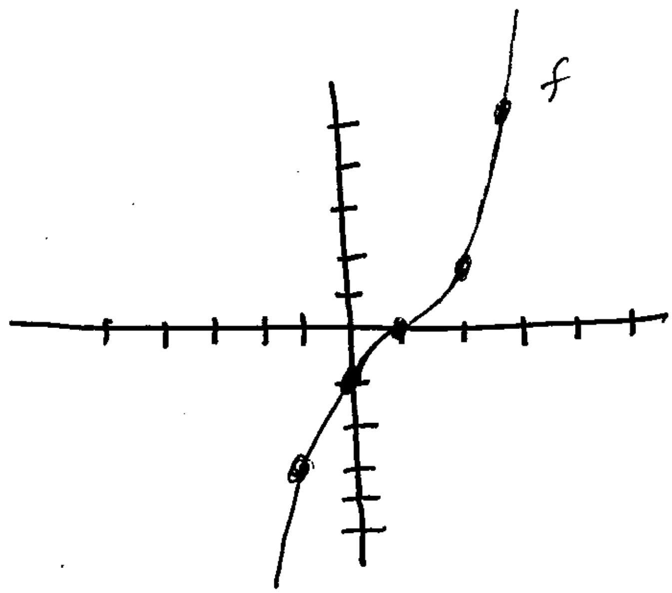
• $g(f(x)) = \dfrac{1}{f(x)-3}$

$$= \frac{1}{\left(\frac{1}{x}+3\right)-3}$$

$$= \frac{1}{\left(\frac{1}{x}\right)} = x$$

Yes, f & g are inverses.

(17) The graph of f is given. Sketch the graph of f^{-1}



SOL