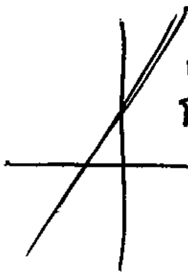
EXAMPLE Find the inverse.

a) $y = 2x + 1$



one-to-one
passes horiz.
line test.

Step 1: Switch x & y

$$x = 2y + 1$$

Step 2: Solve for y.

$$x - 1 = 2y$$

$$\frac{x-1}{2} = y$$

$$y = \frac{x}{2} - \frac{1}{2}$$

$$f^{-1}(x) = \frac{x}{2} - \frac{1}{2}$$

b) $y = \dfrac{x-1}{x+2}$

Step 1: Switch x & y

$$x = \frac{y-1}{y+2}$$

Step 2:

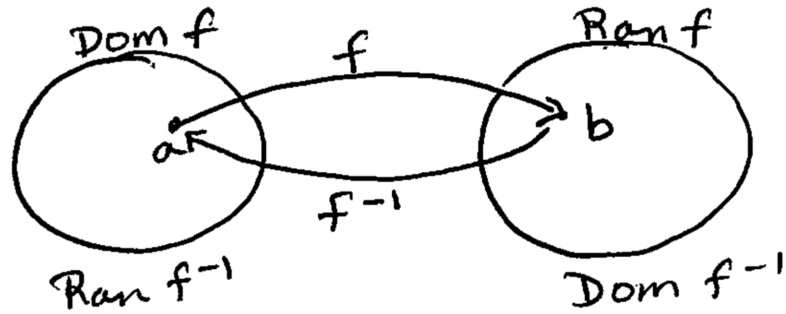
$$x(y+2) = y - 1$$

$$xy + 2x = y - 1$$

$$xy - y = -2x - 1$$

$$y(x-1) = -2x - 1$$

$$y = \frac{-2x-1}{x-1}$$



Cancelation Property

The functions f and g are inverse functions of each other if and only if

$$g(f(x)) = x$$

and

$$f(g(x)) = x$$

EXAMPLE Determine whether $f(x) = x^3 - 1$ and $g(x) = \sqrt[3]{x+1}$ are inverse functions.

SOLUTION

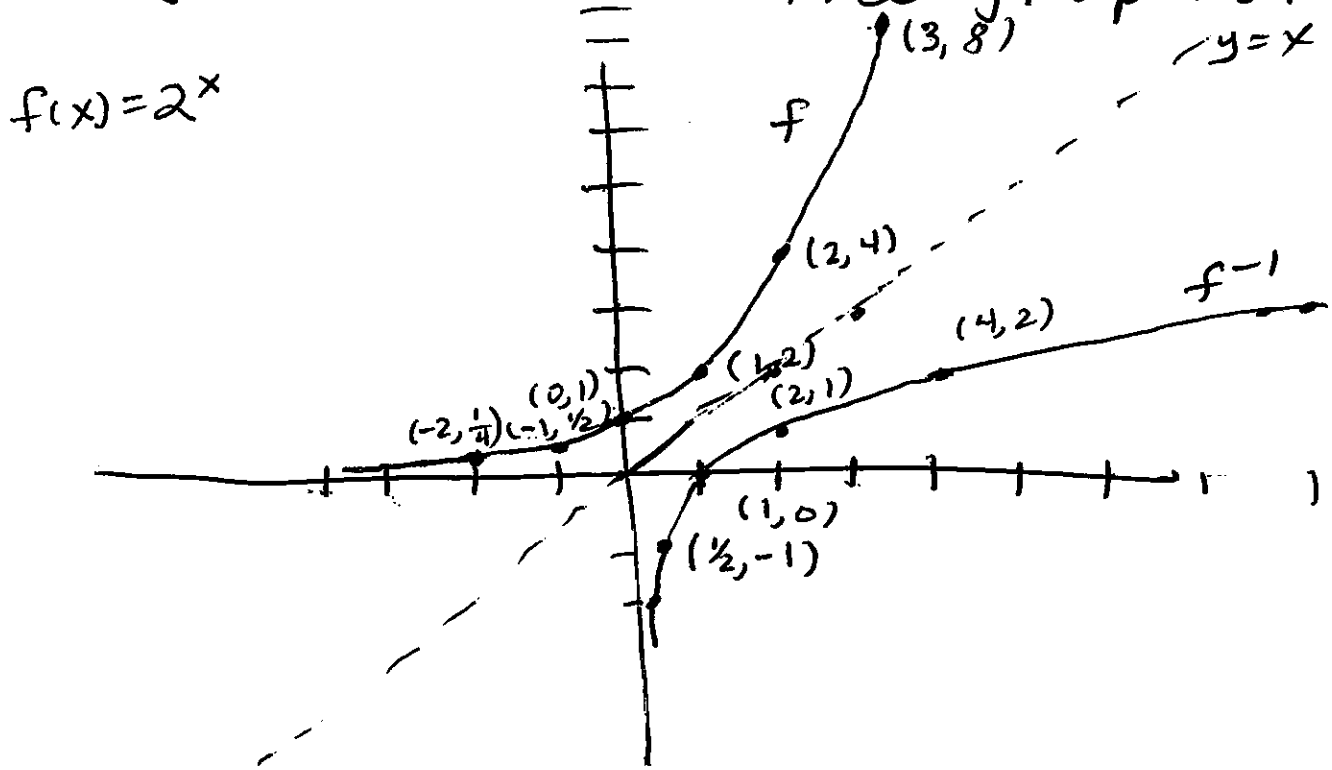
$$\begin{aligned} f(g(x)) &= (g(x))^3 - 1 \\ &= (\sqrt[3]{x+1})^3 - 1 \\ &= (x+1) - 1 \\ &= x \checkmark \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \sqrt[3]{(f(x)) + 1} \\ &= \sqrt[3]{(x^3-1)+1} \\ &= \sqrt[3]{x^3} \\ &= x \checkmark \end{aligned}$$

Yes, f & g are inverses of each other.

The graph of f^{-1} is a reflection about the line $y = x$ of the graph of f.

Example: The graph of f is given. Sketch the graph of f^{-1}.



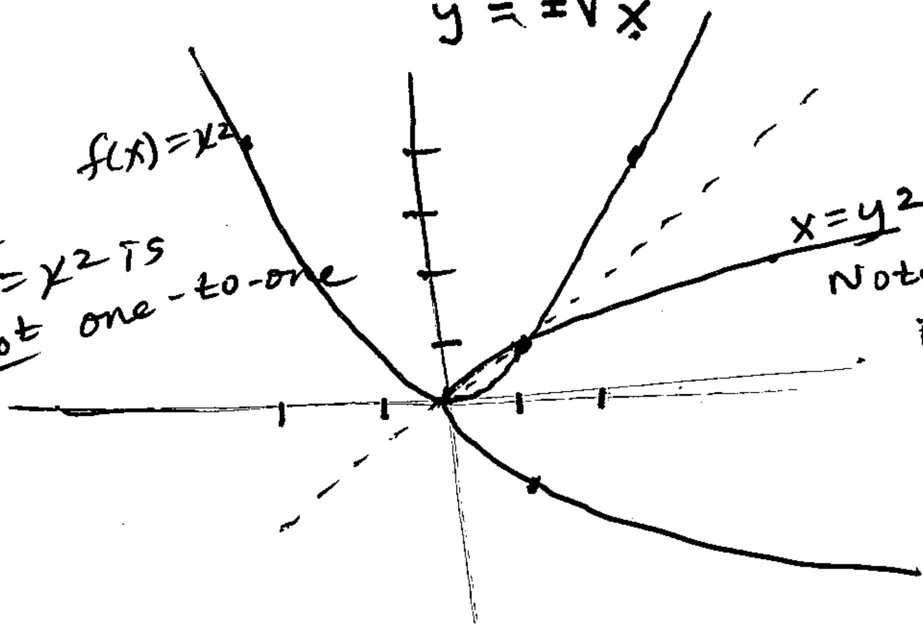
EXAMPLE Let $f(x) = x^2$. Find an inverse relation.

SOL $y = x^2$

Switch x & y.

$x = y^2$

$y = \pm\sqrt{x}$



Note
$f(x) = x^2$ is not one-to-one

Note $x = y^2$ is not a function