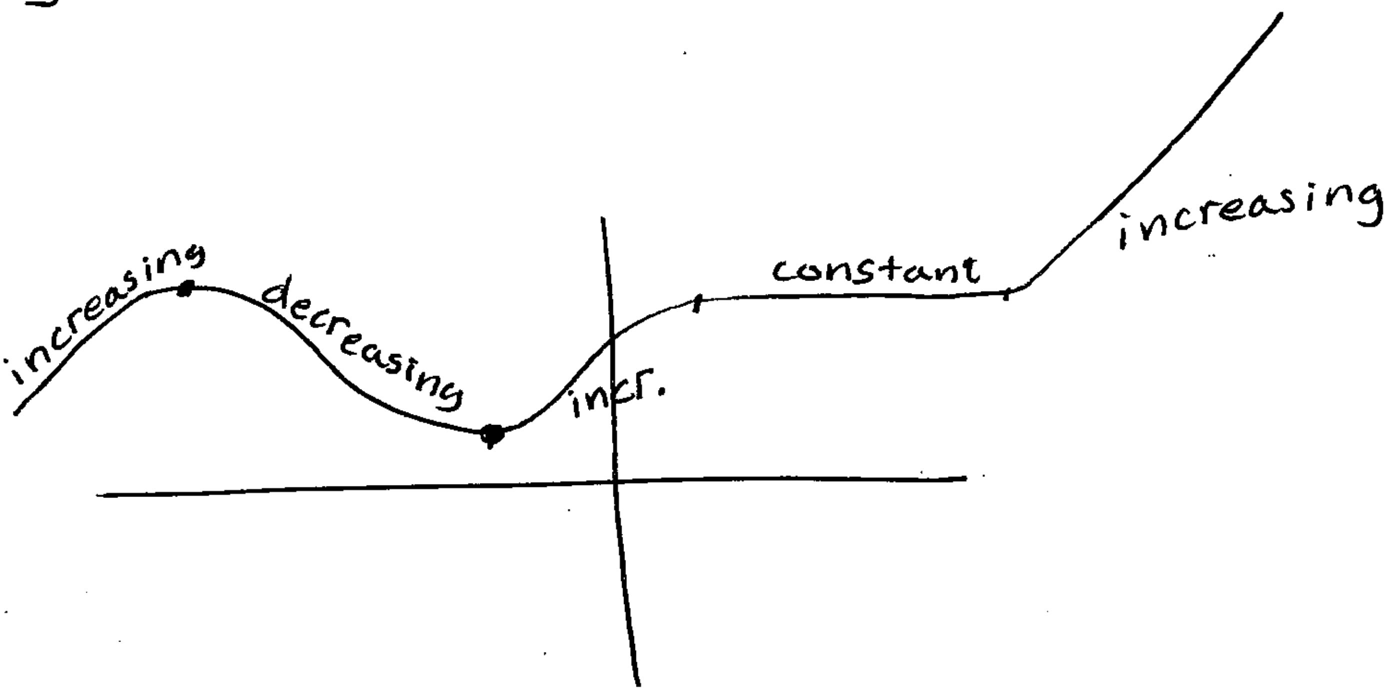
§ 2.2 Notes Continued. Tues. 22-Jun



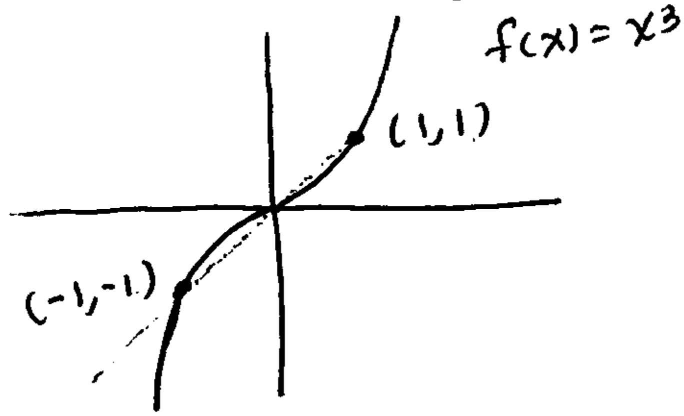
§2.3 Notes Continued.

Odd Functions

- A function f is odd if $f(-x) = -f(x)$

- EXAMPLE: $f(x) = x^3$.
  This is an odd function because
  $$f(-x) = (-x)^3 = -x^3 = -f(x)$$

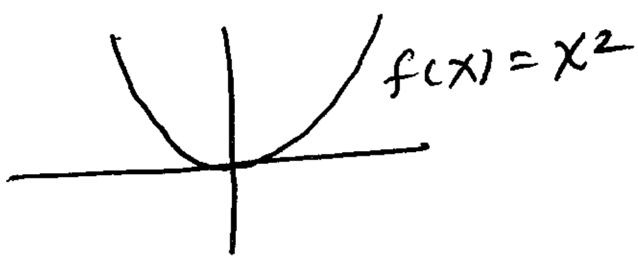
- Odd functions are symmetric about the origin.



- Other examples of odd functions are $y = \sin x$, $y = \tan x$

Even Functions

- A function f is even if
  $f(-x) = f(x)$
- Example: $f(x) = x^2$ is even because
  $f(-x) = (-x)^2 = x^2 = f(x)$
- Even Functions are symmetric about the y-axis



- Other examples of even functions:
  $y = |x|$, $y = \cos x$, $y = x^n$, n is even.

EXAMPLE Determine whether the function is odd, even, or neither.

① $f(x) = 1 + x^2 + x^4$

SOLUTION

$$f(-x) = 1 + (-x)^2 + (-x)^4$$
$$= 1 + x^2 + x^4$$
$$= f(x) \quad \text{even.}$$

② $f(x) = x^3 + x$

SOL

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -(x^3 + x)$$
$$= -f(x) \quad \text{odd}$$

③ $f(x) = x^2 + x^3$

SOL

$$f(-x) = (-x)^2 + (-x)^3$$
$$= x^2 + -x^3$$
$$\neq -f(x) \quad \text{neither}$$
$$\neq f(x)$$

§2.4 Operations with Functions

HW §2.4 # 1-10, 21-28, 35-77 odd

EXAMPLE Let $f(x) = 3x+1$, $g(x) = x^2$

Find

a) $(f+g)(x) = f(x) + g(x)$
$= 3x+1+x^2$
$= x^2+3x+1$

b) $(f-g)(x) = f(x) - g(x) = (3x+1) - (x^2)$
$= -x^2+3x+1$

c) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+1}{x^2}$

d) $(fg)(x) = (3x+1)(x^2)$

Composition of Functions

EXAMPLE: Let $f(x) = 3x+1$, $g(x) = x^2$

Find

a) $f(7) = 3(7)+1 = 21+1 = 22$

b) $g(-3) = (-3)^2 = 9$

c) $f(2+h) = 3(2+h)+1$

d) $f(g(x)) = 3(g(x))+1$
$= 3x^2+1$

e) $g(f(x)) = (f(x))^2 = (3x+1)^2$

The composition of f and g is defined as

$$f \circ g(x) = f(g(x))$$

EXAMPLE: Let $f(x) = \frac{3}{x}$, $g(x) = \sqrt{x}$

Find

a) $f(g(x)) = \frac{3}{(g(x))} = \frac{3}{\sqrt{x}}$

b) $g(f(x)) = \sqrt{(f(x))} = \sqrt{\frac{3}{x}} = \frac{\sqrt{3}}{\sqrt{x}}$

c) $f(f(x)) = \frac{3}{(f(x))} = \frac{3}{(3/x)} = 3 \cdot \frac{x}{3} = x$

d) $g(g(x)) = \sqrt{g(x)} = \sqrt[2]{\sqrt{x}} = (\sqrt{x})^{1/2} = (x^{1/2})^{1/2}$
$= x^{1/4}$
$= \sqrt[4]{x}$

EXAMPLE Write as a composition of $f(x)=\sqrt{x}$, $g(x)=x-3$, $h(x)=2x$.

a) $F(x) = \sqrt{x-3}$ $\qquad$ $F(x) = f(g(x))$

$= \sqrt{g(x)}$ $\qquad$ $F = f \circ g$

$= f(g(x))$

b) $G(x) = x-6$ $\qquad$ $= g(g(x))$

$= (x-3)-3$ $\qquad$ $G = g \circ g$

$= g(x)-3$

c) $H(x) = 2\sqrt{x} - 3$

$= 2f(x) - 3$

$= h(f(x)) - 3$

$= g(h(f(x)))$

$H = g \circ h \circ f$

§2.5 Inverse Functions

#7–33, 39–59, 63–79 odd

EXAMPLE Which graph is a function?

a) $x^2+y^2=1$

No

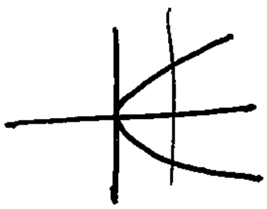
Fails Vert Line Test

b)

$y=f(x)$

Yes. Passes Vert. Line Test

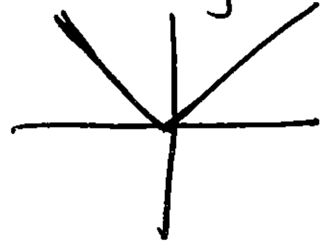
c)



$x=y^2$

No. Fails Vert. Line Test.

d)



$y=|x|$

Yes, passes Vert line test

x	$y=\|x\|$
-2	2
-1	1
0	0
1	1
2	2

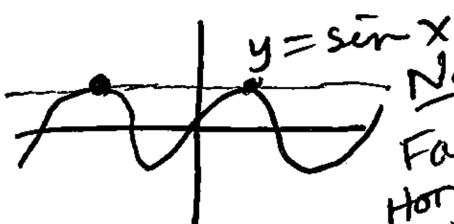
- A function f is <u>one</u>-<u>to</u>-<u>one</u> if for every y-value (value in the range) there exists a <u>unique</u> value x (value in the domain

- f is one-to-one iff $f(a)=f(b)$ implies $a=b$.

• A function f is one-to-one iff it passes the <u>horizontal line test</u>. That is, no horizontal line intersects the graph at more than one point.

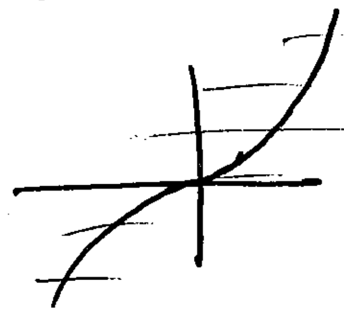
<u>EXAMPLE</u> Which graph represents a one-to-one function?

a)



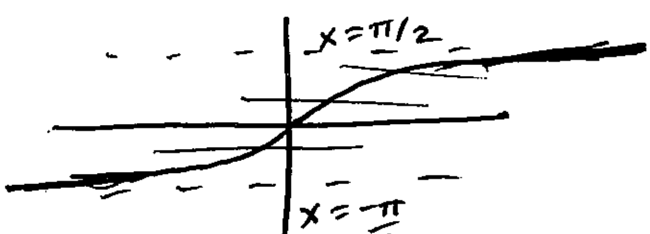
<u>No</u>
Fails Horiz Line Test

b)



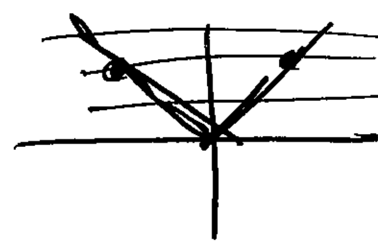
$y = x^3$
Yes.

c) $y = \tan^{-1} x$



Yes

d) $y = |x|$



No

The Inverse of a one-to-one function f is the function f^{-1} where the ordered pairs of f^{-1} are obtained by interchanging the coordinates in each ordered pair of f.

EXAMPLE

$y = x^3$

x	$y = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

The inverse is found by swithing x and y

x	$y = f^{-1}(x)$
-8	-2
-1	-1
0	0
1	1
8	2