Tuesday 22 June 2010

§1.4 Practice Write an equation of the line in $y = mx + b$ form for each line described below.

① The line with slope 3, going through $(1, -2)$.

SOLUTION $m = 3, \; x_1 = 1, \; y_1 = -2$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = 3(x - 1)$$
$$y + 2 = 3x - 3$$
$$\boxed{y = 3x - 5}$$

② The line through $(-2, 3)$ parallel to $y = -\frac{1}{4}x + 5$.

SOLUTION $m = -\frac{1}{4}, \quad x_1 = -2, \; y_1 = 3$

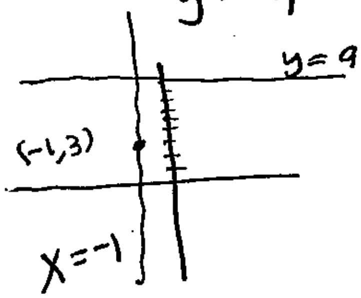
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{1}{4}(x - (-2))$$
$$y - 3 = -\frac{1}{4}x - \frac{1}{2}$$
$$y = -\frac{1}{4}x - \frac{1}{2} + 3$$
$$y = -\frac{1}{4}x - \frac{1}{2} + \frac{6}{2}$$
$$\boxed{y = -\frac{1}{4}x + \frac{5}{2}}$$

③ The line perpendicular to $y = \frac{2}{3}x + 5$ and containing $(2, -3)$.

SOLUTION $m_\perp = \frac{-3}{2}$ (← the neg. reciprocal of $\frac{2}{3}$.)

$$x_1 = 2, \; y_1 = -3$$
$$y - y_1 = m(x - x_1)$$
$$y - (-3) = \frac{-3}{2}(x - 2)$$
$$y + 3 = \frac{-3}{2}x + 3$$
$$\boxed{y = \frac{-3}{2}x}$$

④ The line perpendicular to $y = 9$ and through $(-1, 3)$



SOLUTION

$$\boxed{x = -1}$$

⑤ The line through $(3, 5)$ and $(5, 9)$.

SOLUTION $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{9-5}{5-3} = \dfrac{4}{2} = 2$

$x_1 = 3,\ y_1 = 5$

$y - y_1 = m(x - x_1)$

$y - 5 = 2(x - 3)$

$y - 5 = 2x - 6$

$$\boxed{y = 2x - 1}$$

§1.7 Practice.

1. Write the inequality in interval notation.

a) $0 < x < 3$

$(0, 3)$

b) $-2 \le x < 5$

$[-2, 5)$

c) $2 \le x$ $\quad [2, \infty)$

d) $x \le -3$

$(-\infty, -3]$

e) $x < -1$ or $2 < x$

$(-\infty, -1) \cup (2, \infty)$

2. Solve the inequality.

a) $7 - 5x \le -3$

SOLUTION

$-5x \le -10$

$x \ge \frac{-10}{-5}$

$x \ge 2$ $\qquad [2, \infty)$

b) $\frac{1}{2} - x > \frac{x}{3} + \frac{1}{4}$

Sol Multiply through by lcd.

$12\left(\frac{1}{2} - x\right) > 12\left(\frac{x}{3} + \frac{1}{4}\right)$

$6 - 12x > 4x + 3$

$-12x - 4x > 3 - 6$

$-16x > -3$

$x < \frac{-3}{-16}$

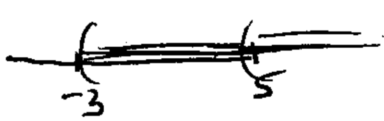
$x < \frac{3}{16}$

$(-\infty, 3/16)$

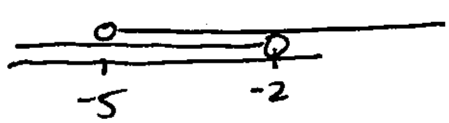
3. Write as a single interval.

a) $(-3, \infty) \cup (5, \infty)$

$= (-3, \infty)$



b) $(-\infty, -2) \cap (-5, \infty) = (-5, -2)$



4. Solve the compound inequality.

a) $1 < 3x - 5 < 7$

$\quad +5 \qquad +5 \qquad +5$

$6 < 3x < 12$

$\frac{6}{3} < x < \frac{12}{3}$

$2 < x < 4$

b) $-2 \le 4 - 6x < 22$

$\quad -4 \qquad -4 \qquad -4$

$-6 \le -6x < 18$

$\frac{-6}{-6} \geq \frac{-6x}{-6} > \frac{18}{-6}$

$1 \geq x > -3$

$-3 < x \le 1$

$(-3, 1]$

5. Solve each absolute value inequality.

a) $|x - 3| < 2$

$-2 < x - 3 < 2$

$+3 \qquad +3 \qquad +3$

$1 < x < 5$

$(1, 5)$

b) $|5 - 4x| \le 1$

$-1 \le 5 - 4x \le 1$

$-6 \le -4x \le -4$

$\frac{-6}{-4} \geq x \geq \frac{-4}{-4}$

$1 \le x \le \frac{3}{2}$

c) $|x - 4| \geq 1$

$x - 4 \le -1$ or $x - 4 \geq 1$

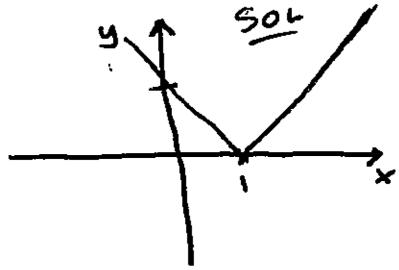
$$\boxed{\begin{array}{c} x \le 3 \quad \text{or} \quad x \geq 5 \\ (-\infty, 3] \cup [5, \infty) \end{array}}$$

§ 2.2, 2.3 Practice.

Sketch the graph. State the domain and range.

① $y = |x-1|$

SOL



Domain: $\mathbb{R}$

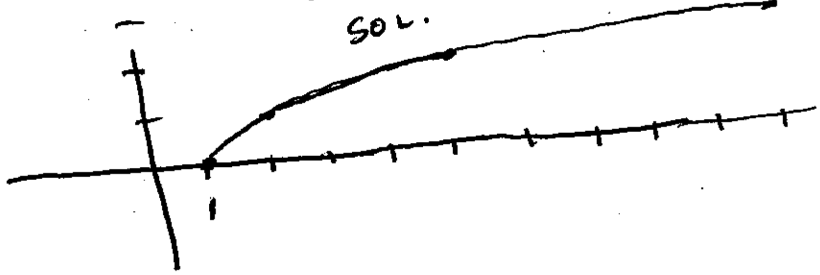
Range: $y \geq 0$

② $y = \sqrt{x-1}$

Domain: $[1, \infty)$

Range: $y \geq 0$

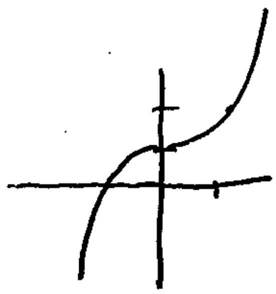
SOL.



③ $y = x^3 + 1$

Dom: $\mathbb{R}$

Ran: $\mathbb{R}$



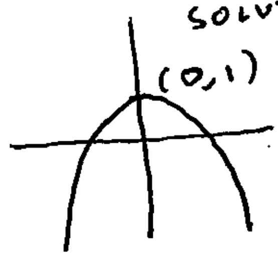
SOL.

④ $y = -x^2 + 1$

Dom: $\mathbb{R}$

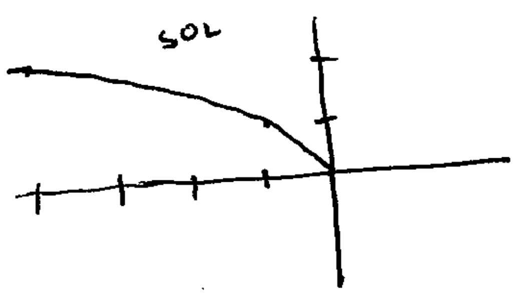
Ran: $y \leq 1$

SOLUTION



⑤ b) $y = \sqrt{-x}$

SOL



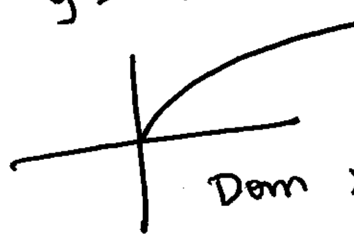
Dom: $x \leq 0$

$(-00, 0]$

Ran $y \geq 0$

5a

$y = \sqrt{x}$



Dom $x \geq 0$

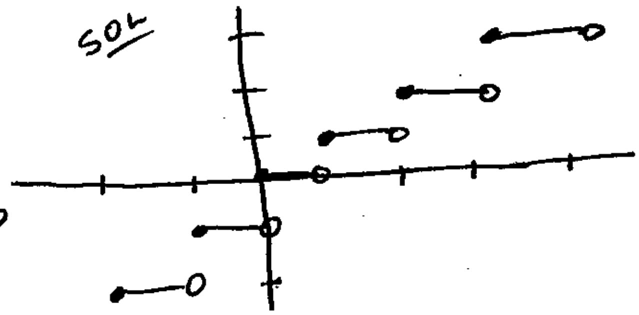
Ran $y \geq 0$

(6) $y = [\![x]\!]$

Dom: $\mathbb{R}$

Ran: All integers $\mathbb{Z}$

SOL

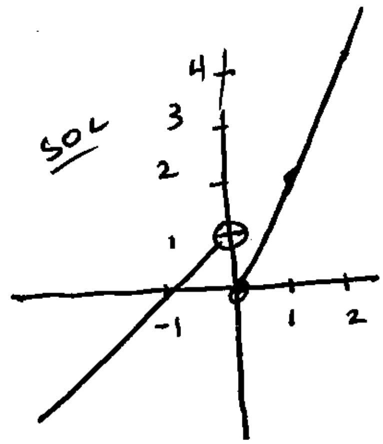


(7) $y = \begin{cases} x+1 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$

Dom: $\mathbb{R}$

Ran: $\mathbb{R}$

SOL

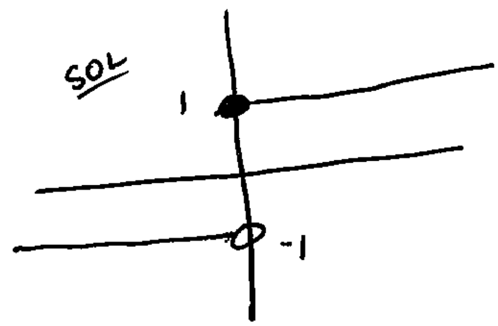


(8) $y = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

Dom: $\mathbb{R}$

Ran: $\{-1, 1\}$

SOL

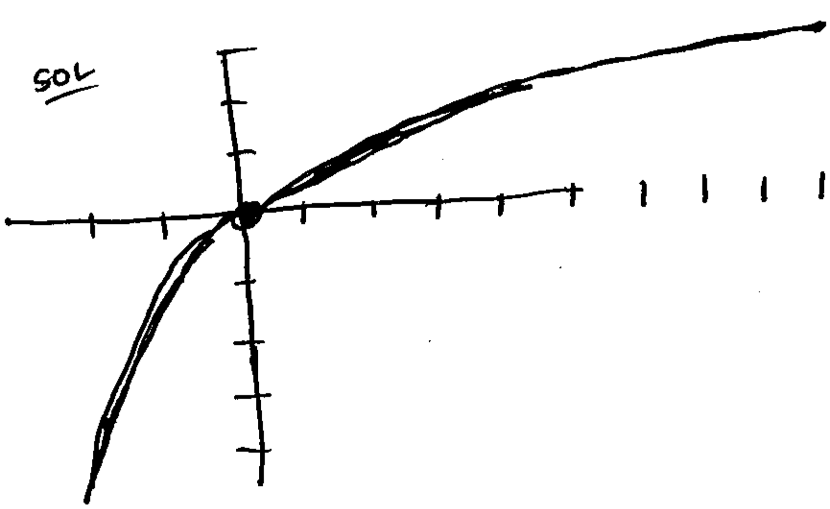


(9) $y = \begin{cases} -x^2 & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$

Dom: $\mathbb{R}$

Ran: $\mathbb{R}$

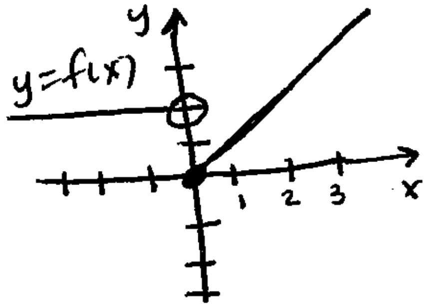
SOL



Write a piece-wise defined function for the graph of each function. State the domain and range.

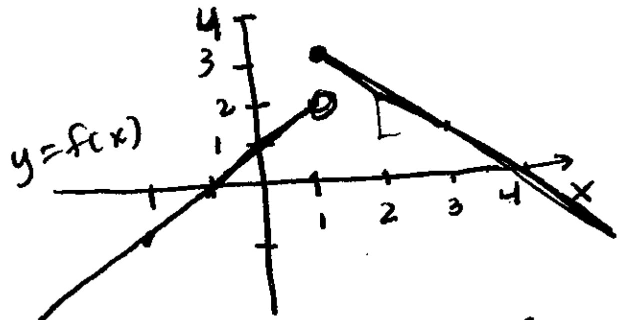
SOLUTION

(10)



$$f(x)=\begin{cases} 1 & \text{if } x<0 \\ x & \text{if } x\geq 0 \end{cases}$$

(11)



SOLUTION

$$f(x)=\begin{cases} x+1 & \text{if } x<1 \\ -x+4 & \text{if } x\geq 1 \end{cases}$$