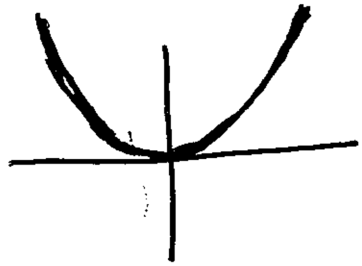
The distance between two real numbers a and b is
$|a-b|$.

§ 2.2 Graphs of Relations and Functions

HW § 2.2 # 1-71 odd

Essential Graphs of Functions

① $y = x^2$

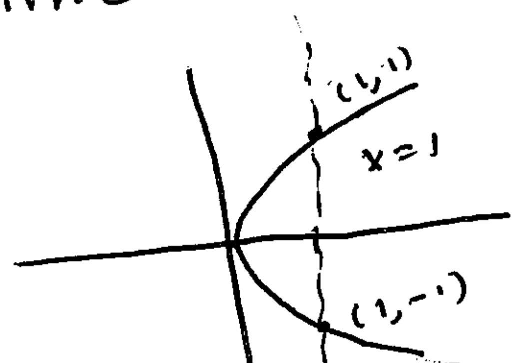


x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

y is a function ~~for x^2 be~~ of x because

- For each x, there exists a unique y value.
- It passes the vertical line test.

② $x = y^2$

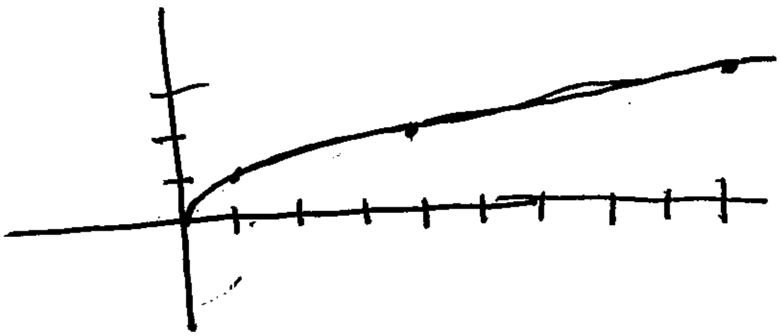


y	$x = y^2$
-2	4
-1	1
0	0
1	1
2	4

x is not a function of y.

- It fails the vertical line test.
- When $x = 1$, $y = 1$ or $y = -1$.

③ $y = \sqrt{x}$



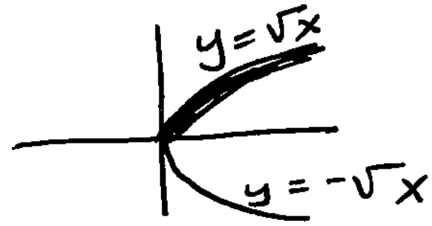
x	$y=\sqrt{x}$
0	0
1	1
4	2
9	3

<u>Note</u>: $x = y^2$ is equivalent to

$$y = \pm\sqrt{x}$$

$y=\sqrt{x}$ is the upper part,

$y = -\sqrt{x}$ is bottom half.



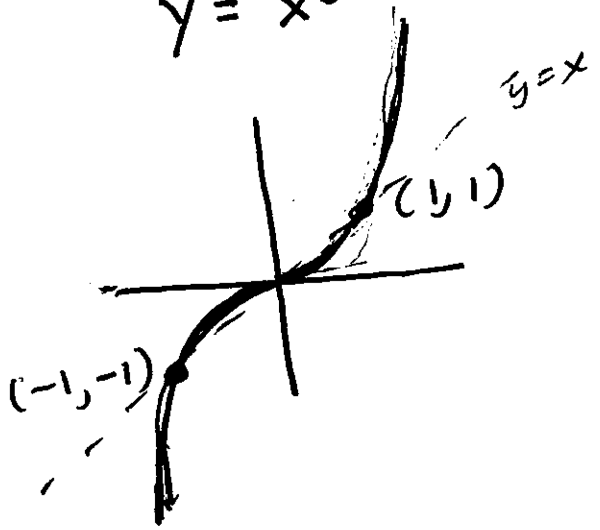
The <u>domain</u> of a function, $y=f(x)$, is the set of all possible x values.

The <u>range</u> is the set of all possible y-values.

The Domain of $y=\sqrt{x}$ is $\{x \mid x \geq 0\}$

④ The cubic function

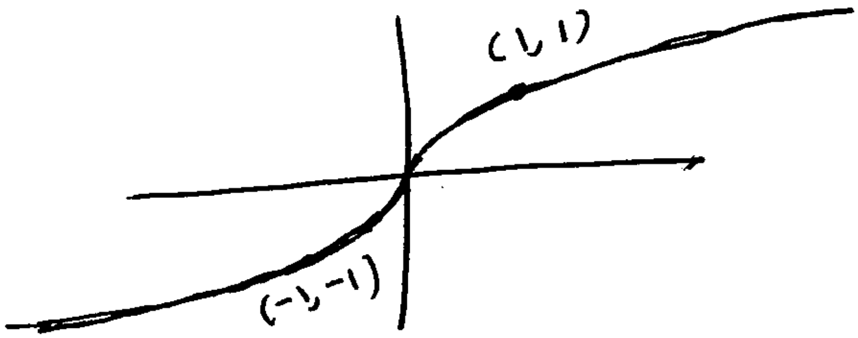
$$y = x^3$$



x	$y = x^3$
-2	$(-2)^3 = -8$
-1	$(-1)^3 = -1$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$

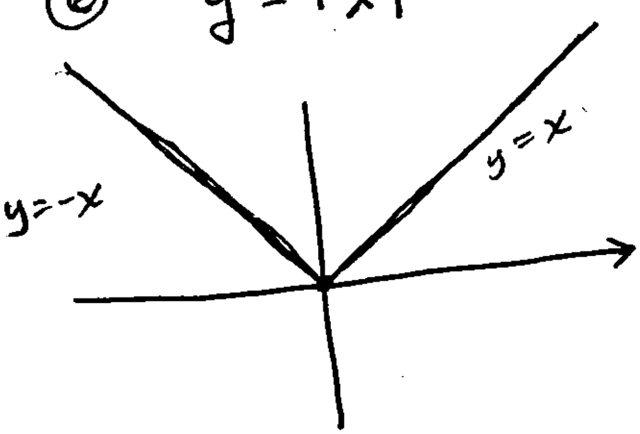
⑤ The ~~cube~~ cubic root

$$y = \sqrt[3]{x}$$



x	y
0	0
1	$\sqrt[3]{1} = 1$
8	$\sqrt[3]{8} = 2$

⑥ $y = |x|$

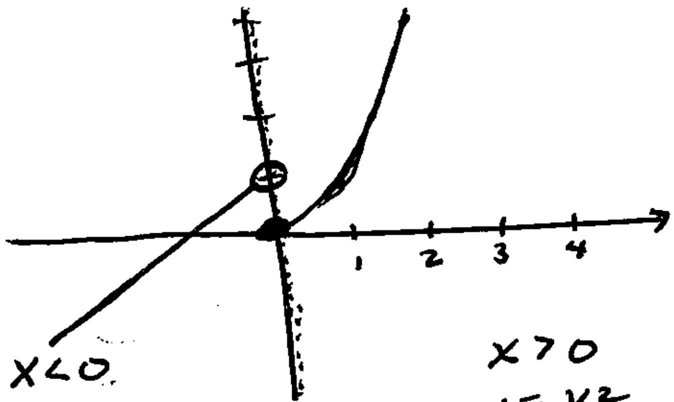


$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Piece-wise defined functions.

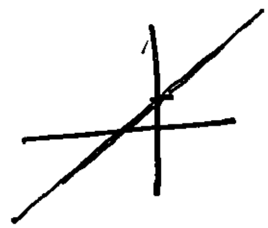
(7) $$f(x) = \begin{cases} x^2 & \text{when } x \geq 0 \\ x+1 & \text{when } x < 0 \end{cases}$$

$y = x^2$



$y = x+1$

$m = 1,\ b = 1$

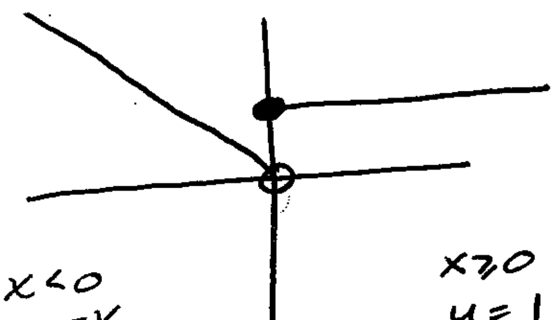


$x < 0$
$y = x+1$

$x > 0$
$y = x^2$

~~What is the~~

(8) $$f(x) = \begin{cases} -x & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$$



$x < 0$
$y = -x$
$m = -1,\ b = 0$

$x \geq 0$
$y = 1$

what is the range of this function? $0 < y$

$$\text{Ran} f = \{y \mid 0 < y\}$$

The Greatest Integer Function

$y = [[x]]$

is the greatest integer less than or equal to x.

(The set of integers is $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$)

<u>Evaluate:</u>

$[[2]] = 2$

$[[2.1]] = 2$

$[[2.2]] = 2$

$[[2.3]] = 2$

$\vdots$

$[[2.9]] = 2$

$[[3]] = 3$

for $2 \le x < 3$

$y = [[x]] = 2$

$[[3.1]] = 3$

$[[3.2]] = 3$

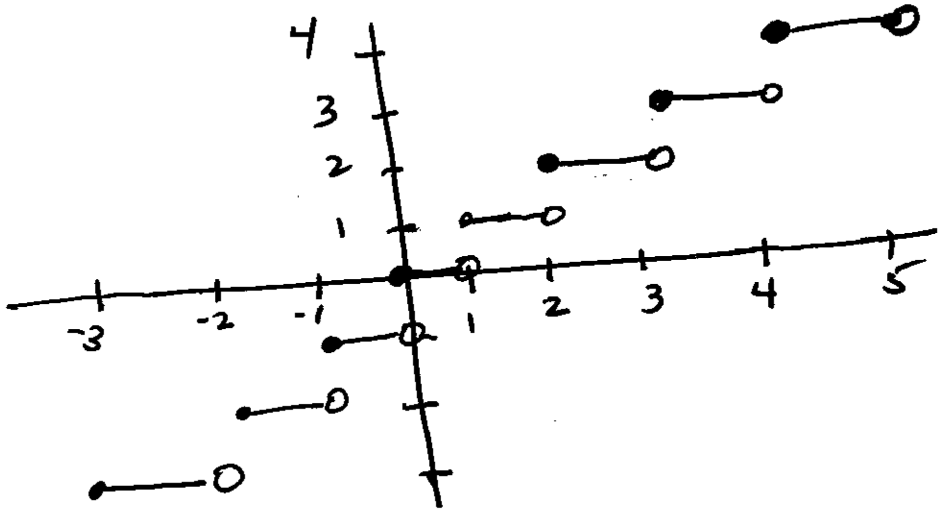
$\vdots$

$[[3.9]] = 3$

$[[-2.1]] = -3$

$[[-2.5]] = -3$

$[[-\pi]] = -4$



§ 2.3 Translations

HW § 2.3 # 1-89 odd.

We begin with the graph of $y = f(x)$.

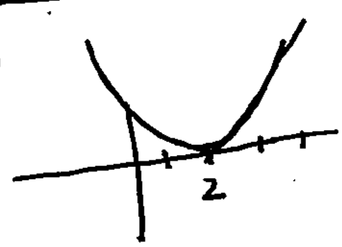
if $c > 0$

- $y = f(x-c)$ <u>shifts</u> the graph to the <u>right by c</u>
- $y = f(x+c)$ <u>shifts</u> the graph <u>left</u> <u>by c</u>.

<u>Example</u>: sketch the graph.

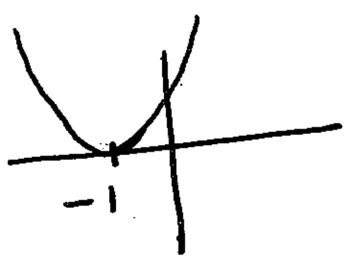
a) $y = x^2$

b) $y = (x-2)^2$



c) $y = (x+1)^2$

x	y
-2	$(-2+1)^2 = 1$
-1	$(-1+1)^2 = 0$
0	$(0+1)^2 = 1$

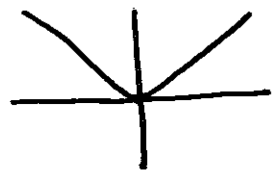


For $c > 0$

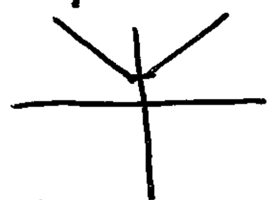
- $y = f(x) + c$ shifts the graph up by c.
- $y = f(x) - c$ shifts the graph down by c.

<u>Example</u>: sketch the graph.

① $y = |x|$



② $y = |x| + 1$

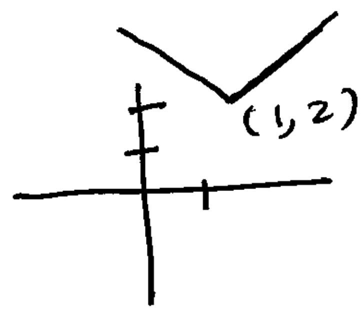


x	$y = \|x\| + 1$
0	$\|0\| + 1$
-1	$\|-1\| + 1 = 2$
1	$\|1\| + 1 = 2$

③ $y = |x| - 2$

④ $y = |x - 1| + 2$

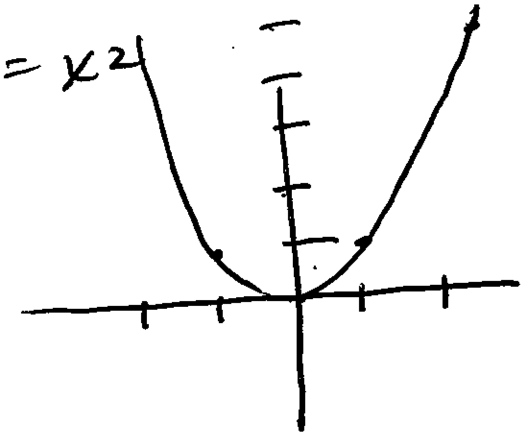
right by 1   up by 2



Suppose $c > 1$,

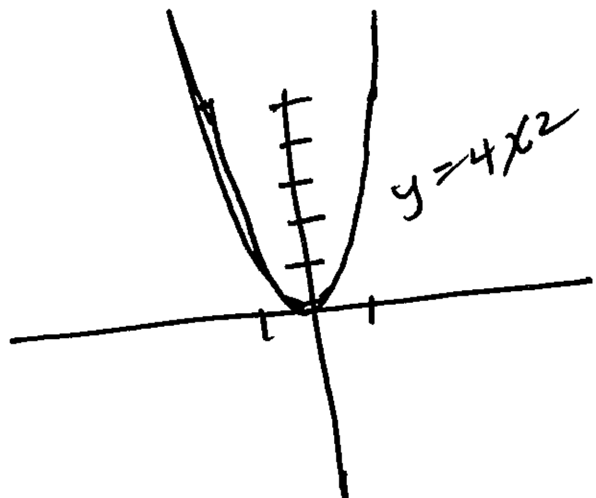
then $y = c\,f(x)$ is stretched vertically by a factor of c.

Example: ① $y = x^2$



x	$y = x^2$
-1	1
0	0
1	1

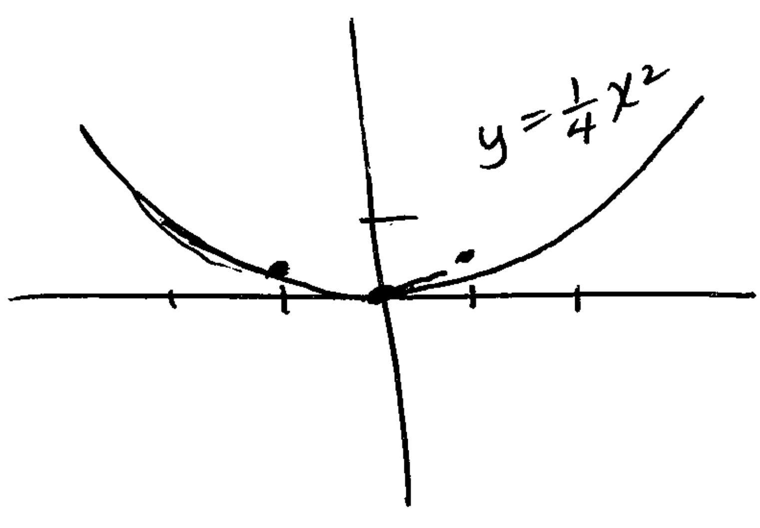
② $y = 4x^2$



x	$y = 4x^2$
-1	$4(-1)^2 = 4$
0	$4(0)^2 = 0$
1	$4(1)^2 = 4$

Suppose $0 < c < 1$, then $y = c\,f(x)$ is compressed by a factor of c.

③ $y = \frac{1}{4}x^2$

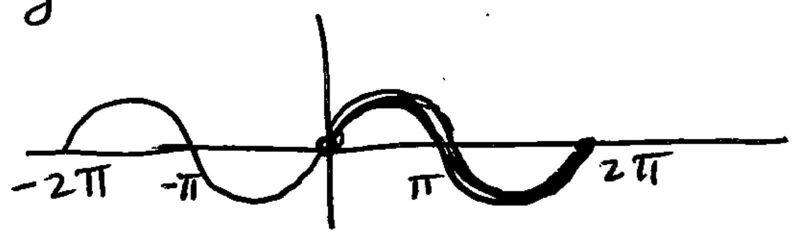


x	$y = \frac{1}{4}x^2$
-2	$\frac{1}{4}(-2)^2 = 1$
-1	$\frac{1}{4}(-1)^2 = \frac{1}{4}$
0	$\frac{1}{4}(0)^2 = 0$
1	$\frac{1}{4}(1)^2 = \frac{1}{4}$
2	$\frac{1}{4}(-2)^2 = 1$

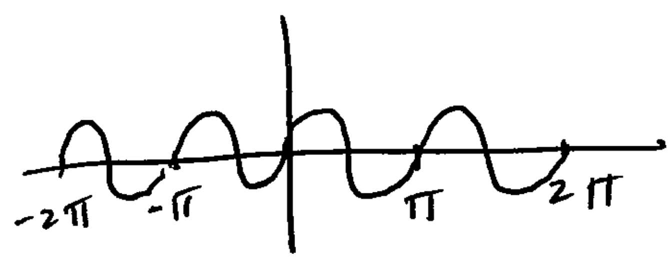
- If $c > 1$
  $y = f(cx)$ is compressed horizontally by c.
- If $0 < c < 1$, $y = f(cx)$ is stretched by c.

Example:

- $y = \sin x$ Period $P = 2\pi$



- $y = \sin 2x$ $P = \frac{2\pi}{2} = \pi$



- $y = \sin\left(\frac{x}{2}\right)$ $P = \frac{2\pi}{1/2} = 2\pi \cdot \frac{2}{1}$
  $= 4\pi$