§1.4 Lines

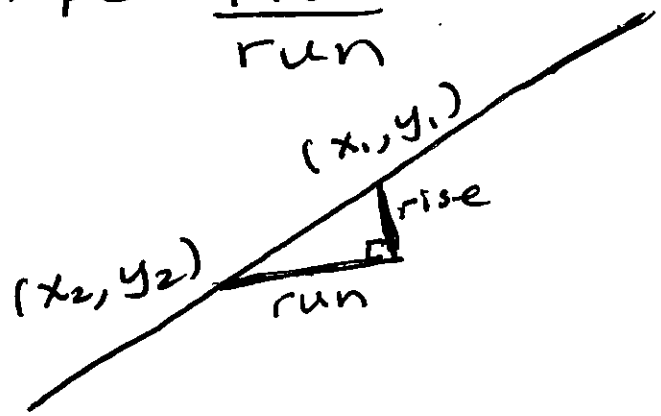
~~Mondo~~ Monday

HW §1.4 #11-16, 25-35 odd

21-Jun

The slope of a line

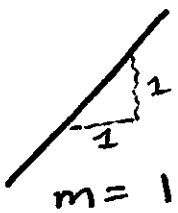
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

EXAMPLE Estimate the slope.

a)



$m = 1$

b)

$m = 2$

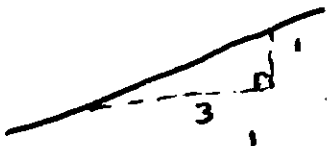
c)

$m = 10$

d)

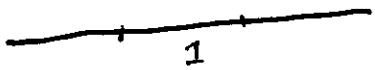
$m = \frac{1}{0}$ undefined

e)



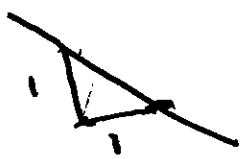
$m = \frac{1}{3}$

f)



$m = \frac{0}{1} = 0$

g)



$m = \frac{-1}{1}$

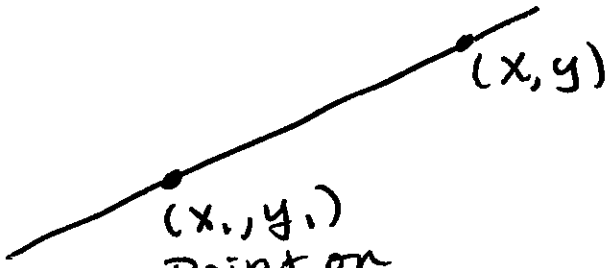
$m = -1$

h)

$m = -3$

Point slope Formula

Monday 21- Jun



$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1$$

$$\boxed{y - y_1 = m(x - x_1)}$$

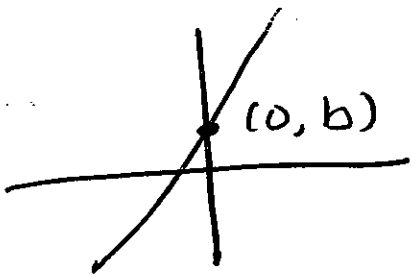
Slope-intercept Formula.

If $(0, b)$ is the y-intercept, and m is the slope of a line, we get

$$y - b = m(x - 0)$$

$$y - b = mx$$

$$\boxed{y = mx + b}$$



$x_1 = 0, y_1 = b$

EXAMPLE Find the equation of the line that passes through $(-2, 5)$ and $(5, -1)$.

SOLUTION

Slope: $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - (-1)}{-2 - (5)} = \frac{6}{-7} = \frac{-6}{7}$

$y - y_1 = m(x - x_1)$ $\quad m = -\frac{6}{7},\ x_1 = -2,\ y_1 = 5$

$y - 5 = -\frac{6}{7}(x - -2)$

$y - 5 = -\frac{6}{7}(x + 2)$

$y - 5 = -\frac{6}{7}x - \frac{12}{7}$

$y = -\frac{6}{7}x - \frac{12}{7} + 5$

$y = -\frac{6}{7}x - \frac{12}{7} + \frac{35}{7}$

$$\boxed{y = -\frac{6}{7}x + \frac{23}{7}}$$

OR this way.

$y = mx + b, \quad m = -\frac{6}{7}$

$y = -\frac{6}{7}x + b$

find b.
let $x = -2,\ y = 5$

$5 = -\frac{6}{7}(-2) + b$

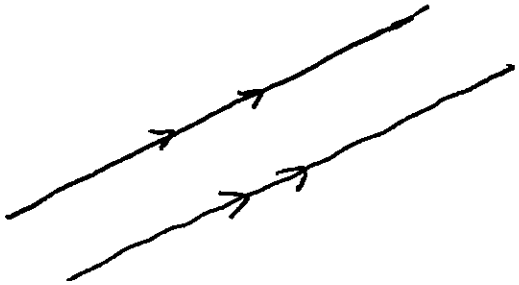
$5 = \frac{12}{7} + b$

$5 - \frac{12}{7} = b$

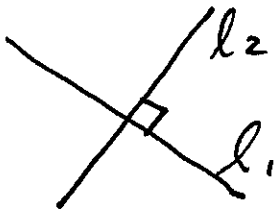
$\frac{35}{7} - \frac{12}{7} = b, \quad b = \frac{23}{7}$

$$\boxed{y = -\frac{6}{7}x + \frac{23}{7}}$$

- Parallel lines have equal slope.



- Perpendicular ~~Perp~~ Lines l_1 and l_2 with slopes m_1 and m_2 satisfy

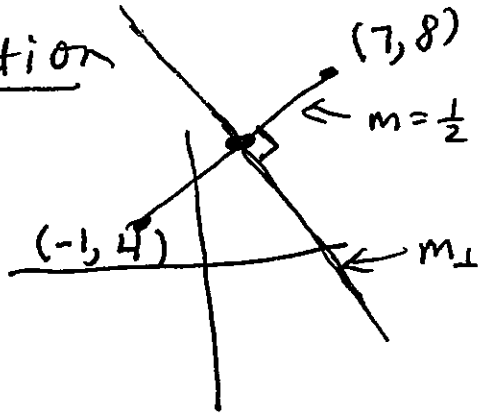


$$m_1 m_2 = -1$$

$$m_2 = -\frac{1}{m_1}$$

EXAMPLE: Find the perpendicular bisector of the line segment with endpoints $(-1, 4)$ and $(7, 8)$

Solution



- Midpoint of the line segment, $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{x_1 + x_2}{2} = \frac{-1+7}{2} = \frac{6}{2} = 3$$

$$\bar{y} = \frac{8+4}{2} = \frac{12}{2} = 6$$

Midpoint $(3, 6)$

Slope of line segment.

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$m = \frac{8-4}{7--1} = \frac{4}{8} = \frac{1}{2}$

Slope of $\perp$ bisector:

$m_\perp = \frac{-2}{1} = -2$

Equation of $\perp$ bisector.

Point $(3, 6)$, $m_\perp = -2$

$y - y_1 = m(x - x_1)$

$y - 6 = -2(x - 3)$

$y - 6 = -2x + 6$

$y = -2x + 6 + 6$

$\boxed{y = -2x + 12}$

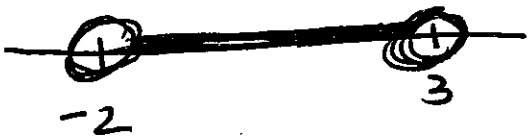
§1.7 Linear and Absolute Inequalities

HW §1.7 #1-77 odd

Interval Notation

EXAMPLE: Write in interval notation. Sketch the graph.

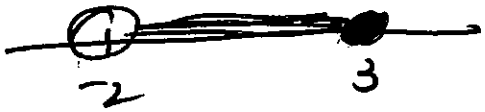
a) $-2 < x < 3$



$(-2, 3)$

or

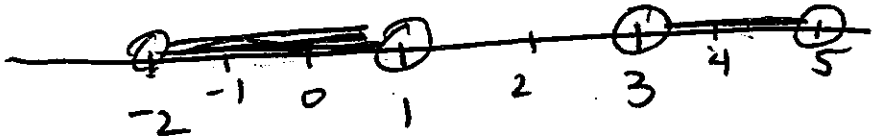
b) $-2 < x \leq 3$ $\quad (-2, 3]$



c) $2 < x$ $\quad (2, \infty)$

d) $x \leq 2$ $\quad (-\infty, 2]$

e) $-2 < x < 1$ or $3 < x < 5$

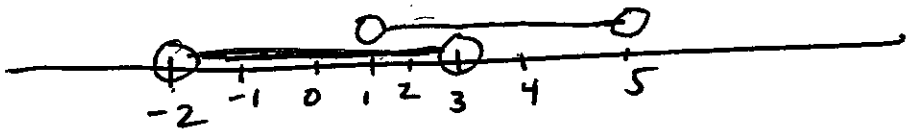


$(-2, 1) \cup (3, 5)$

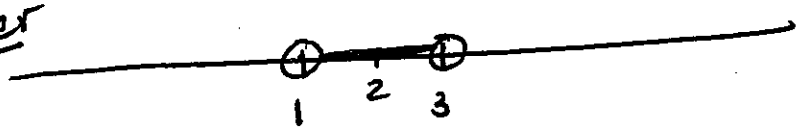
↑ union

"or" corresponds to union.

f) $-2 < x < 3$ and $1 < x < 5$



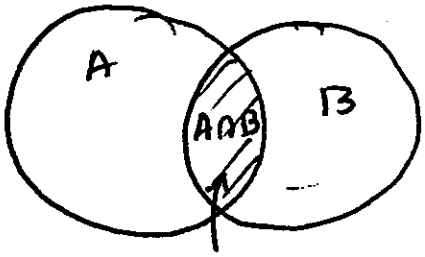
Answer



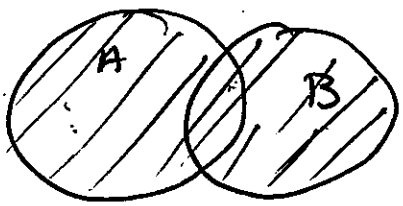
$1 < x < 3$

$(1, 3)$

"and" corresponds to intersection $\cap$



$A \cap B$



$A \cup B$

Solving Linear Inequalities

Example: Find all x that satisfy the inequality.

a) $-3x - 9 < 0$

$-3x < 9$

$x > \frac{9}{-3}$

$x > -3$

$-3 < x$

$(-3, \infty)$

When multiplying or ~~dividing~~ dividing by a negative, we change the direction of the inequality.

b) A compound inequality.

$-14 < 3x - 5 < 4$

$+5 \quad +5 \quad +5$

$-9 < 3x < 9$

$\frac{-9}{3} < x < \frac{9}{3}$

$-3 < x < 3$

This means $-3 < x$ and $x < 3$

c) $\frac{4}{-4} - 3x < \frac{-2}{-4}$ or $3(x-2) \leq -6$

$-3x < -6$

$x > \frac{-6}{-3}$

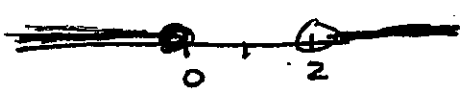
$x > 2$

$(x-2) \leq \frac{-6}{3}$

$x - 2 \leq -2$

$+2 \quad +2$

$x \leq 0$



$x > 2$ or $x \leq 0$

We can write this as $(-\infty, 0] \cup (2, \infty)$

Do not write. $2 < x \leq 0$ this implies that 2 is less than 0.

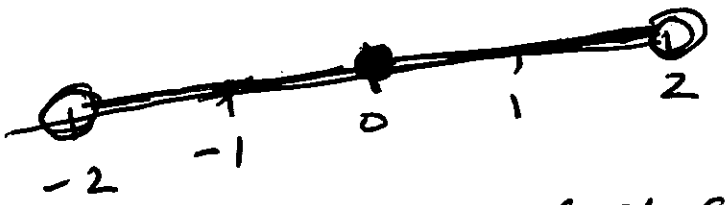
Absolute Value Inequalities

$|x| < a$ means $-a < x < a$

Example: Solve. Write in interval notation

(1) $|x| < 2$

$-2 < x < 2$



The set of all x such that $|x| < 2$ is the set of points whose distance from the origin is less than 2.

(2) $3|x-5| < 9$

$|x-5| < 3$

$-3 < x-5 < 3$

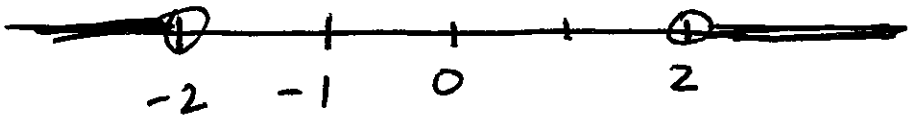
$+5 \quad +5 \quad +5$

$\boxed{2 < x < 8}$

The interval $(2, 8)$

The set $\{x \mid 2 < x < 8\}$

③ $|x| > 2$



$2 < x$
or
$x < -2$

$|x| > a$ means
$x < -a$ or $a < x$

$x < -2$ or $2 < x$

~~$2 < x < -2$~~

④ $3|x+5| > 6$

$|x+5| > \frac{6}{3}$

$|x+5| > 2$

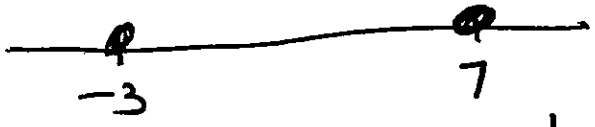
$x+5 < -2$ or $2 < x+5$

$x < -7$ or $-3 < x$

$(-\infty, -7) \cup (-3, \infty)$

Extra: Find the distance between the ~~points~~ ~~one~~ real numbers on the number line.

a) $-3, 7$



$|7-(-3)| = 10$ or $|-3-7| = |-10| = 10$

b) a and 5.

$|a-5|$ or $|5-a|$