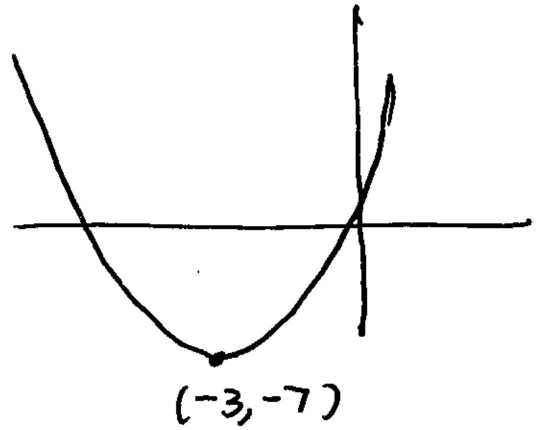
§3.1 HW SOLUTIONS

① $y = x^2 + 6x + 2$

$y = (x^2 + 6x + 9) - 9 + 2$

$y = (x+3)^2 - 7$

vertex $(-3, -7)$

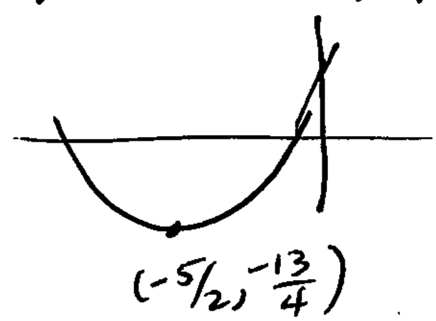


② $y = x^2 + 5x + 3$

$y = \left(x^2 + 5x + \left(\frac{5}{2}\right)^2\right) - \left(\frac{5}{2}\right)^2 + 3$

$y = (x + 5/2)^2 - \frac{25}{4} + \frac{12}{4}$

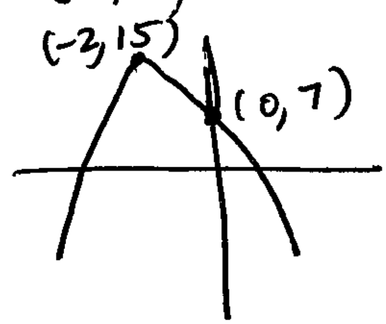
$y = (x + 5/2)^2 - \frac{13}{4}$ vertex $\left(-5/2, \frac{-13}{4}\right)$



③ $y = -2x^2 - 8x + 7$

$y = -2(x^2 + 4x + 4) + 8 + 7$

$y = -2(x+2)^2 + 15$



④ $y = x^2 + 4x + 3$

$y = (x^2 + 4x + 4) - 4 + 3$

$y = (x+2)^2 - 1$

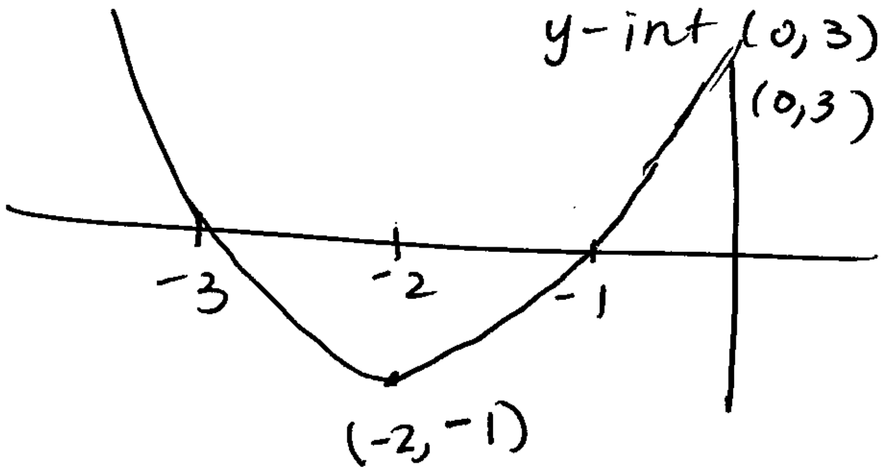
vertex $(-2, -1)$

<u>x-int</u> $x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$(-1, 0), (-3, 0)$

y-int $(0, 3)$



⑤ $y = x^2 - 2x - 8$

$y = (x^2 - 2x + 1) - 1 - 8$

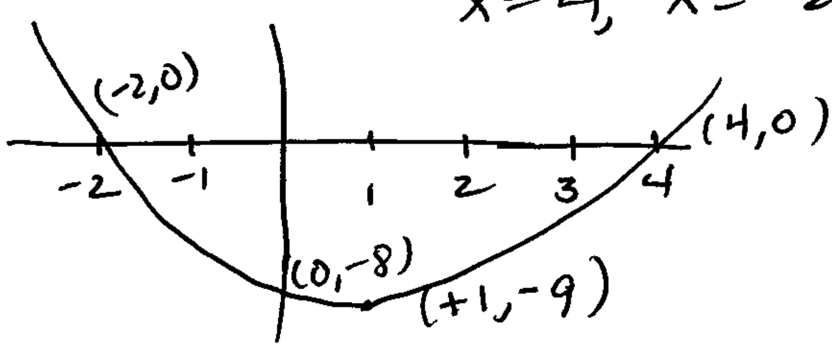
$y = (x-1)^2 - 9$

vertex $(1, -9)$

x-int. $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, \quad x = -2 \qquad (+4, 0), (-2, 0)$



⑥ $y = 2x^2 + 6x + 4$

$y = 2\left(x^2 + 3x + (3/2)^2\right) - 2\left(\frac{3}{2}\right)^2 + 4$

$y = 2(x + 3/2)^2 - \frac{9}{2} + 4$

$y = 2(x + 3/2)^2 - \frac{1}{2}$

vertex $(-3/2, -1/2)$

x-int $2x^2 + 6x + 4 = 0$

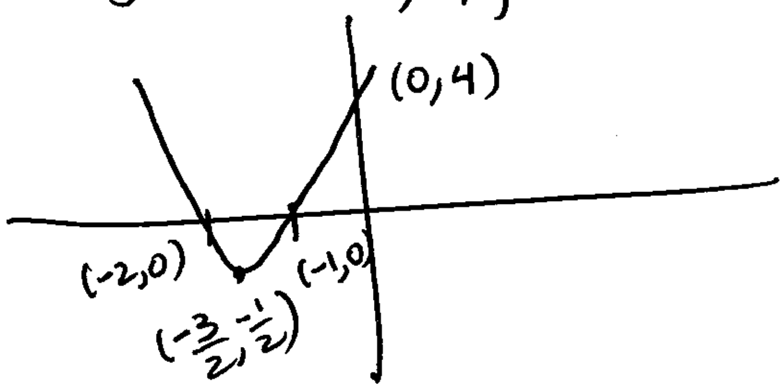
$2(x^2 + 3x + 2) = 0$

$2(x+1)(x+2) = 0$

$x = -1, \; x = -2$

$(-1,0), (-2,0)$

y-int $(0, 4)$



⑦ $x^2-5x-6<0$

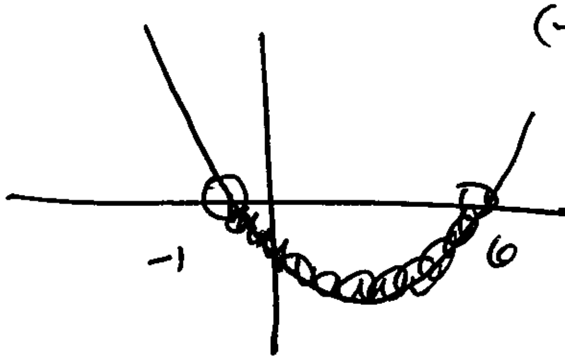
sketch $y=x^2-5x-6$

<u>x-int</u> $x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=-1,\ x=6$

$(-1,0),\ (6,0)$



Answer: $(-1,6)$

⑧

$x^2+3x+1>0$

$y=x^2+3x+1$

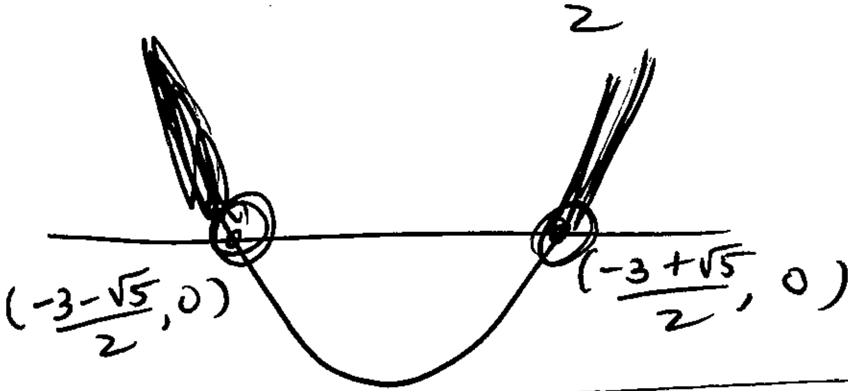
<u>zeros</u>

$$x=\frac{-3\pm\sqrt{3^2-4\cdot1\cdot1}}{2\cdot1}$$

$$=\frac{-3\pm\sqrt{5}}{2}$$

$\left(\frac{-3-\sqrt{5}}{2},0\right)$

$\left(\frac{-3+\sqrt{5}}{2},0\right)$



$$\boxed{\left(-\infty,\ \frac{-3-\sqrt{5}}{2}\right)\cup\left(\frac{-3+\sqrt{5}}{2},\ \infty\right)}$$

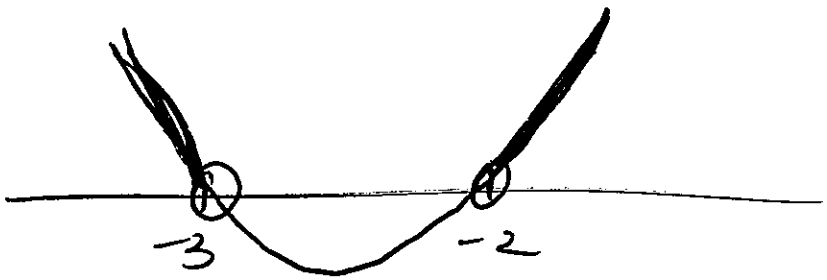
(9) $x^2+5x+6>0$

$y=x^2+5x+6$

<u>zeros</u> $x^2+5x+6=0$

$(x+3)(x+2)=0$

$(-3,0),\ (-2,0)$



$(-\infty,-3)\cup(-2,\infty)$

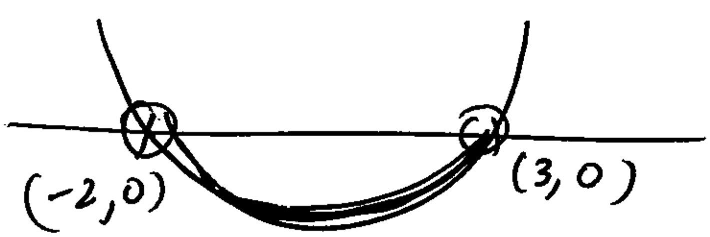
(10) $x^2-x-6<0$

$y=x^2-x-6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$(3,0),\ (-2,0)$



Answer $(-2,3)$

§3.1 # 11 A ball is thrown straight up into the air.

The height is given by

$$y(t) = -16t^2 + 32t$$

y in feet, t seconds.

what is the maximum height?

<u>SOLUTION</u> Find the vertex.

The graph is a parabola.

$$h = \frac{-b}{2a} = \frac{-32}{2(-16)} = 1 \text{ sec}$$

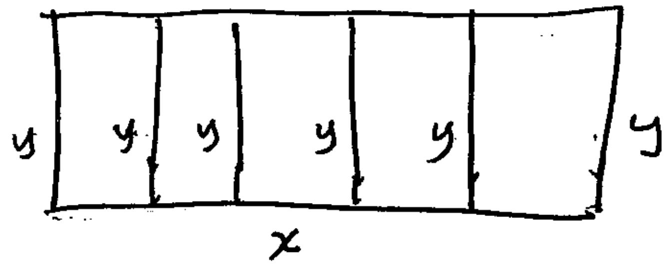
$$y(1) = -16(1)^2 + 32(1) = 16 \text{ feet}$$

<u>Answer 16 ft</u>.

§3.1

#12

2400 ft of fencing.



What dimensions maximize overall area?

<u>SOLUTIONS</u>

<u>FORMULAS</u>

$$2x + 6y = 2400$$

$$A = xy$$

<u>Eliminate a variable</u>

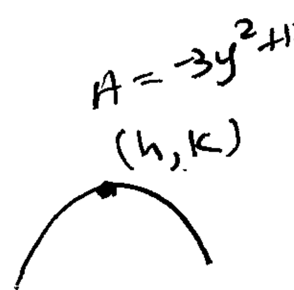
$$2x + 6y = 2400$$

$$2x = -6y + 2400$$

$$x = -3y + 1200$$

$$A = xy = (-3y + 1200)y$$

$$A = -3y^2 + 1200y$$



<u>Find h</u> : $h = \frac{-b}{2a} = \frac{-1200}{2(-3)} = \underline{200f}$

$$\boxed{y = 200\text{ ft}}$$

<u>Find x</u>

$$2x + 6y = 2400$$

$$2x + 6(200) = 2400$$

$$2x = 1200$$

$$\boxed{x = 600\text{ ft}}$$

Find the point of intersection of the parabola and the straight line given by the following equations. Sketch the graph of the line and the parabola on the same plane.

13. $y = -x + 6$ and $y = x^2 - 2x$

SOLUTION

POINTS OF INTERSECTION:

$$\begin{aligned} y &= -x + 6 \\ y &= x^2 - 2x \\ x^2 - 2x &= -x + 6 \\ x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x = 3 & \quad\quad x = -2 \end{aligned}$$

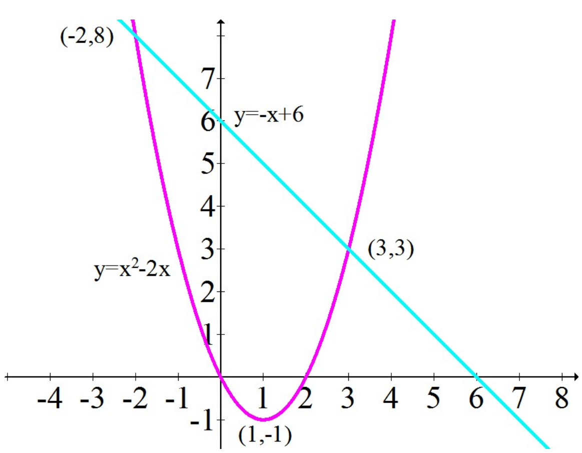
When $x = 3$, $y = -(3) + 6 = 3$. Point of intersection $(3, 3)$.
When $x = -2$, $y = -(-2) + 6 = 8$. Point of intersection $(-2, 8)$.

The function $y = x^2 - 2x$ can be written in $y = a(x-h)^2 + k$ form.

$$y = (x^2 - 2x + 1) - 1 = (x-1)^2 - 1$$

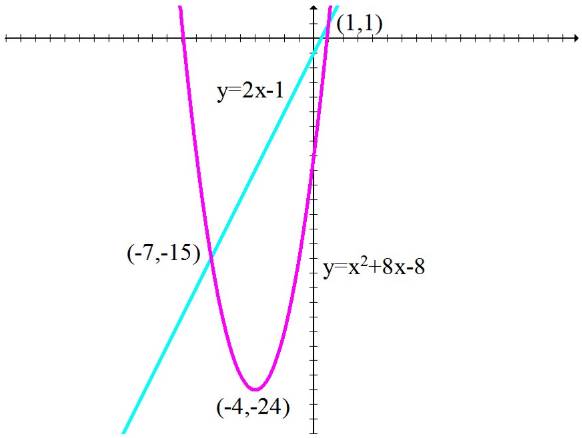
The vertex is $(1, 1)$.

The x-intercepts of $y = x^2 - 2x$ are found by solving $x^2 - 2x = 0$. We get $x(x-2) = 0$; $x = 0$ or $x = 2$.



14. $y = 2x - 1$ and $x^2 + 8x - 8$

    SOLUTION Points of intersection: $(-7, -15)$, $(1, 1)$



15. $y = -x + 1$ and $-x^2 - 3x + 4$

    SOLUTION Points of intersection: $(-3, 4)$, $(1, 0)$