3.2 Zeros of Polynomial

3.5 Graphs of Polynomials

1. What is the degree of the following polynomials?

   (a) $3x^2 - 4x + 5$

   (b) $-5x^5 + 5$

   (c) $-38x^4 + x^3 - x - 1$

   (d) $(3x^2 - 4x + 5)(-5x^4 + x)$

   (e) $(-5x^5 + x)(-7x + 3)$

   (f) $(-4x^2 + 5x - 4)(3x^3 + x - 1)$

   (g) $(6x^7 - x^3 + 5)(7x^4 - 3x^2 + x - 1)$

2. Find the polynomial $q(x)$ and $r(x)$ of the Euclidean algorithm when $f(x) = 4x^3 - x + 2$, and:

   (a) $g(x) = x - 2$

   (b) $g(x) = x^2 - 1$

   (c) $g(x) = x^2 + 1$

   (d) $g(x) = x^2 - x$

   (e) $g(x) = x^2 - x + 1$

   (f) $g(x) = x^2 + x - 1$

   (g) $g(x) = x^3 + 2$

   (h) $g(x) = x^3 - x + 1$

3. Show that the given number c is a root of the given polynomial f, then factor f completely over the real numbers, if it is possible.

   (a) $f(x) = 2x^3 - 3x^2 - 8x - 3, \quad c = -1$

   (b) $f(x) = x^3 - 3x^2 - 18x + 40$, $c = 2$

(c) $f(x) = 6x^3 + 3x^2 - 39x + 18$, $c = -3$

(d) $f(x) = 3x^3 + 12x^2 - 12x - 3$, $c = 1$

(e) $f(x) = 4x^3 - 4x^2 - 44x - 40$, $c = -2$

4. Factor the fourth degree polynomial by writing it as a quadratic. Then find all real and imaginary solutions.

   (a) $x^4 - 5x^2 + 4 = 0$
   (b) $x^4 - 13x^2 + 36 = 0$
   (c) $x^4 - 7x^2 + 10 = 0$
   (d) $x^4 - 1 = 0$
   (e) $x^4 - 3x^2 - 4 = 0$

5. For the given polynomial functions,

   - Find the zeros and their multiplicity.
   - Find the lead term.
   - Sketch the graph.

   (a) $y = (x-1)^2(x+2)^3$
   (b) $y = (x+1)^2x^2(x-2)$
   (c) $y = -(x-1)^2(x+1)^2$
   (d) $y = (x-2)^3(x+1)^2$
   (e) $y = x^2(x-3)(x+2)$