

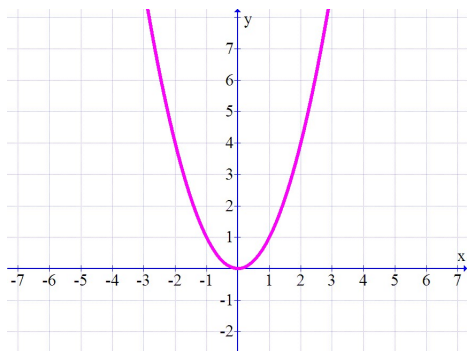
### 3.5 The Graphs of Polynomials

We will now try to sketch the graphs of polynomial functions. We are going to look at polynomial functions that are factored completely.

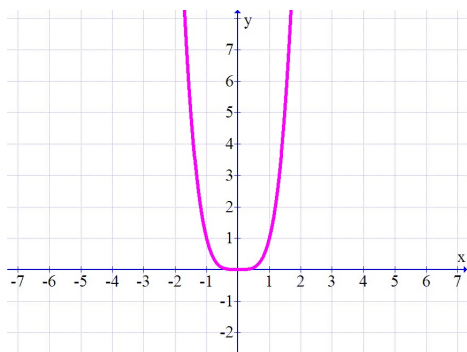
We begin by studying power functions, that is, functions of the form  $y = x^n$ . We have already studied the quadratic  $y = x^2$  and the cubic function  $y = x^3$ .

Let's look at the graph of  $y = x^n$  when  $n$  is odd.

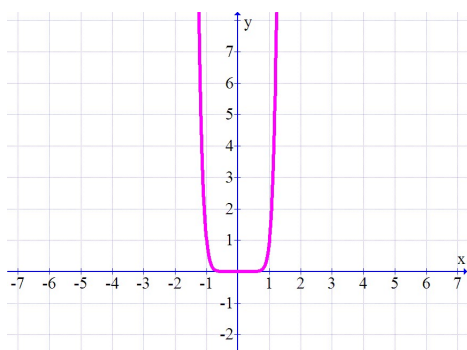
- $y = x^2$



- $y = x^4$



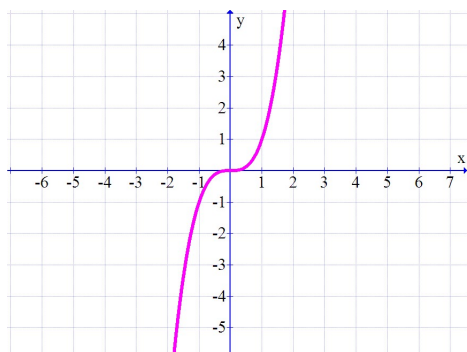
- $y = x^{10}$



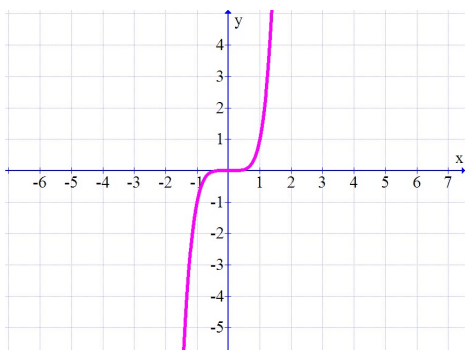
If  $n$  is even, then the graph of  $y = x^n$  looks roughly like the graph of  $y = x^2$ . When  $n$  is even, we see that as  $x$  becomes a very large positive or negative number,  $f(x)$  becomes a very large positive number. So as the graph moves to the far right or to the far left, it will start going up through the roof.

Now let's study the graph of  $y = x^n$  when  $n$  is odd.

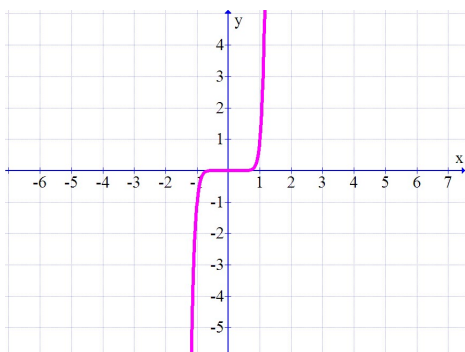
- $y = x^3$



- $y = x^5$



- $y = x^{11}$

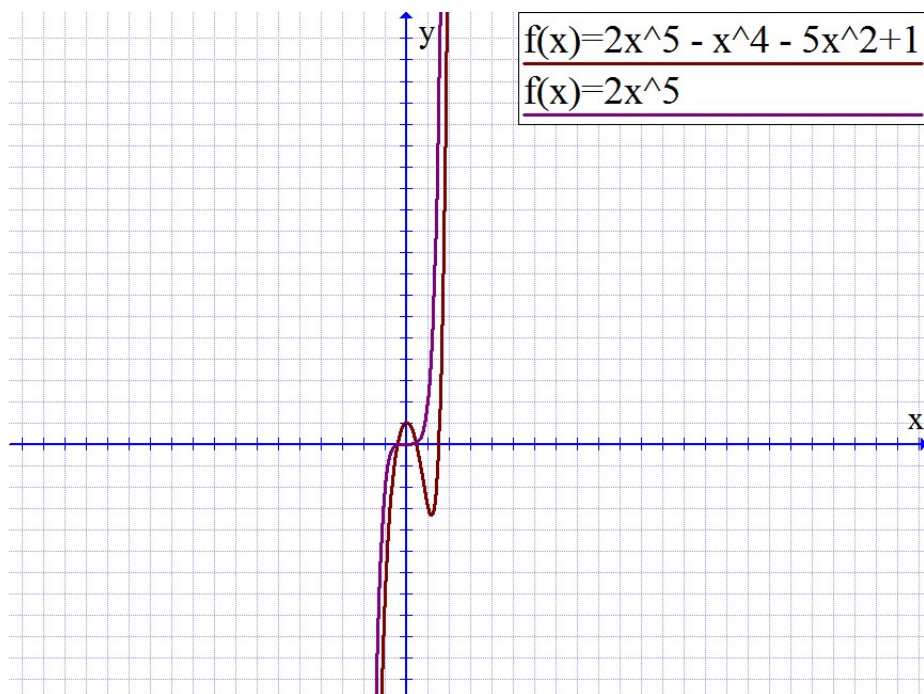


If  $n$  is odd, then the graph of  $y = x^n$  looks roughly like the graph of  $y = x^3$ . When  $n$  is odd, we see that as  $x$  becomes a very large positive number,  $f(x)$  becomes a very large positive number. As the graph of  $f$  goes to the far right, it goes up through the roof. As  $x$  becomes a large negative number,  $f(x)$  becomes a large negative number. As the graph of  $f$  goes to the far left, it goes down through the floor.

We have the following useful principle for general polynomial functions.

*For large values of  $x$ , a polynomial will behave like its lead term.*

For example, let's compare the graphs of the functions  $f(x) = 2x^5 - x^4 - 5x^2 + 1$  and the graph of  $y = 2x^5$ . We see that for large values of  $x$ , the functions look very similar.



We now try to make a rough sketch of a polynomial function that is in a factored form.

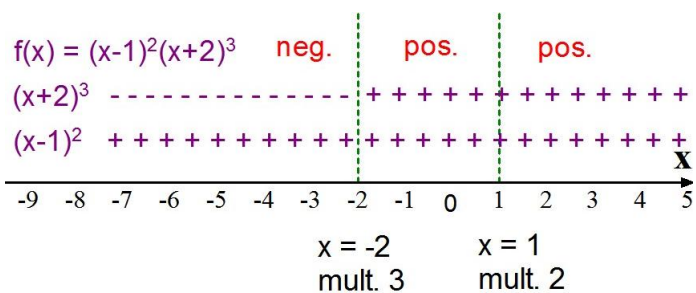
**Example.** Let  $f(x) = (x - 1)^2(x + 2)^3$ .

1. Find the **zeros** of  $f$  and state their **multiplicity**.

**SOLUTION**  $x = 1$  and  $x = -2$ . The **multiplicity** of a zero is the degree of the corresponding factor. Therefore  $x = 1$  has multiplicity 2 because it corresponds with the factor  $(x - 1)^2$ , and the degree of this factor is 2. The zero  $x = -2$  has multiplicity 3 because it corresponds with the factor  $(x + 2)^3$ , and the degree of this factor is 3.

2. Make a sign chart showing where  $f$  is positive or negative.

**SOLUTION**

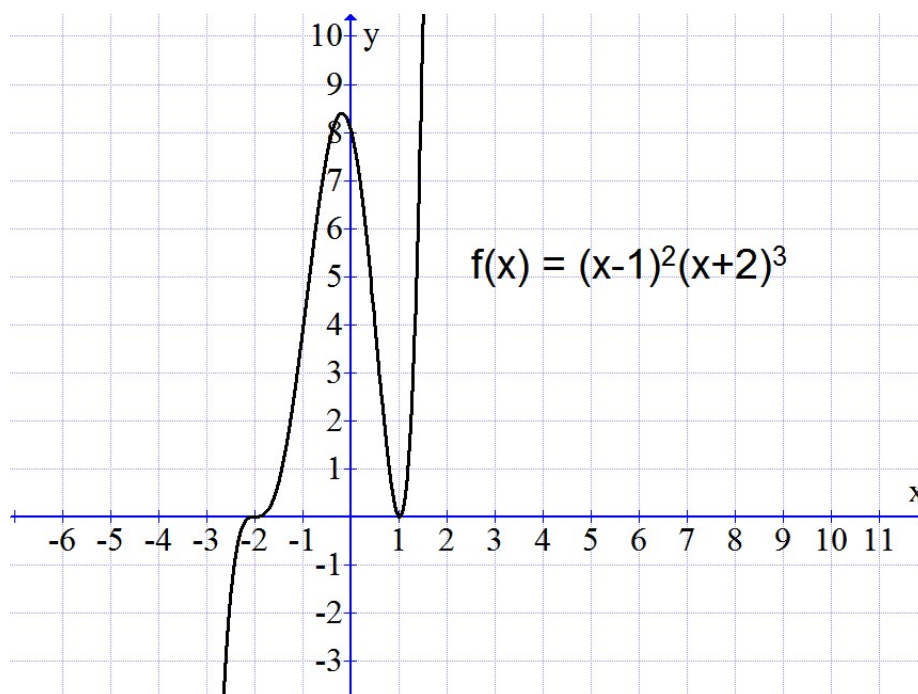


Note that the sign of  $f$  does not change as  $x$  passes over the zero with even multiplicity, while the sign of  $f$  does change as it passes over a zero with odd multiplicity.

3. State the degree of the polynomial and give its leading term.

**SOLUTION** If we were to expand  $f(x) = (x-1)^2(x+2)^3$ , we would see that the term of highest degree, i.e. the lead term, would be  $x^2x^3 = x^5$ . The graph of  $y = x^5$  roughly looks like a parabola. For large values of  $x$ , the graph of  $f$  will be close to the graph of  $y = x^5$ .

4. Sketch the graph.



Note that the graph touches the  $x$ -axis at zeros of even multiplicity and passes through the  $x$ -axis at zeros of odd multiplicity.