Math 124 Chapter 5 Practice Test

Tuesday, 6 April 2010

1. §5.1 Find the remaining five trigonometric functions of θ.

$$\csc\theta = -\frac{5}{2},\ \theta \text{ in quadrant III}$$

$\sin\theta = -\frac{2}{5}$ $\qquad$ $\csc\theta = \frac{-5}{2}$

$\cos\theta = -\frac{\sqrt{21}}{5}$ $\qquad$ $\sec\theta = \frac{-5}{\sqrt{21}} = -5\frac{\sqrt{21}}{21}$

$\tan\theta = +\frac{2\sqrt{21}}{21}$ $\qquad$ $\cot\theta = +\ \frac{\sqrt{21}}{2}$

- $\sin\theta = \frac{-2}{5}$, find $\cos\theta$

$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{-2}{5}\right)^2$$
$$= 1 - \frac{4}{25} = \frac{25}{25} - \frac{4}{25}$$
$$\cos^2\theta = \frac{21}{25}$$
$$\cos\theta = -\frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2/5}{\frac{-\sqrt{21}}{5}} = \frac{-2}{5}\cdot\frac{-5}{\sqrt{21}} = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

2. §5.2 Verify that the trigonometric equation is an identity.

$$\frac{1-\cos\theta}{1+\cos\theta} = (\cot\theta - \csc\theta)^2$$

LHS

$$\frac{1-\cos\theta}{1+\cos\theta}$$

$$= \left(\frac{1-\cos\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= (\csc\theta - \cot\theta)^2$$

$$= (\cot\theta - \csc\theta)^2$$

equals RHS

- $\cos(u-v) = \cos u \cos v + \sin u \sin v$
- $\cos(u+v) = \cos u \cos v - \sin u \sin v$
- $\sin(u+v) = \sin u \cos v + \cos u \sin v$
- $\sin(u-v) = \sin u \cos v - \cos u \sin v$

3. Find the exact value of the following expressions.

(a) $\cos \frac{11\pi}{12}$. $= \cos\left(\frac{2}{12}\pi + \frac{9}{12}\pi\right) = \cos\left(\frac{\pi}{6} + \frac{3}{4}\pi\right)$

$= \cos\frac{\pi}{6}\cos\frac{3}{4}\pi - \sin\frac{\pi}{6}\sin\frac{3}{4}\pi$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

(b) $\sin(165°) = \sin(120° + 45°)$

$= \sin 120° \cos 45° + \cos 120° \sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

4. If α and β are acute angles such that $\sin\alpha = -4/5$ and $\cos\beta = 12/13$, with α in quadrant III and β in quadrant IV, find

Find $\cos\alpha$

$\sin\alpha = \frac{-4}{5}$

$\cos^2\alpha = 1 - \sin^2\alpha$

$= 1 - (-\frac{4}{5})^2$

$= \frac{25}{25} - \frac{16}{25} = \frac{9}{25}$

$\cos\alpha = -\frac{3}{5}$

QIII

(a) $\sin(\alpha - \beta)$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= (-\frac{4}{5})(\frac{12}{13}) - (\frac{-3}{5})(\frac{-5}{13})$

$= \frac{-48}{65} - \frac{15}{65} = \frac{-63}{65}$

Find $\sin\beta$

$\cos\beta = \frac{12}{13}$

$\sin^2\beta = 1 - \cos^2\beta$

$= 1 - (\frac{12}{13})^2$

$= \frac{169}{169} - \frac{144}{169}$

$\sin^2\beta = \frac{25}{169}$

$\sin\beta = -\frac{5}{13}$

neg QIV

(b) $\cos(\alpha - \beta)$ $= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= (-\frac{3}{5})(\frac{12}{13}) + (-\frac{4}{5})(\frac{-5}{13})$

$= \frac{-36}{65} + \frac{20}{65} = -\frac{16}{65}$

(c) State the quadrant in which $(\alpha-\beta)$ ~~$\alpha+\beta$~~ lies.

$\sin(\alpha-\beta)$ is neg

$\cos(\alpha-\beta)$ is neg

So $\alpha-\beta$ is in Quad III

$\cos\theta = \frac{3}{4}$. Find $\sin\theta$.
θ in Q1

$\sin^2\theta = 1 - \cos^2\theta$
$= 1 - (3/4)^2$
$= \frac{16}{16} - \frac{9}{16} = \frac{7}{16}$

$\sin\theta = \frac{\sqrt{7}}{4}$

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

5. Suppose that $\cos \cancel{2}\theta = \frac{3}{4}$, and that θ is an acute angle. Find the following values.

   (a) $\sin 2\theta$ $= 2\sin\theta\cos\theta$
   $= 2\left(\frac{\sqrt{7}}{4}\right)\left(\frac{3}{4}\right) = \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}$

   (b) $\cos 2\theta$ $= \cos^2\theta - \sin^2\theta$
   $= \left(\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2 = \frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \frac{1}{8}$

   (c) $\tan 2\theta$ $= \frac{\sin 2\theta}{\cos 2\theta} = \frac{3\sqrt{7}}{8} \Big/ \frac{1}{8} = \frac{3\sqrt{7}}{8} \cdot \frac{8}{1}$
   $= \boxed{3\sqrt{7}}$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \quad \sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \quad \tan\frac{A}{2} = \frac{\sin A}{1+\cos A} \qquad \tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

6. Find each of the following.

(a) $\sin\frac{x}{2}$ given that $\cos x = -\frac{5}{8}$, with $\frac{\pi}{2} < x < \pi$.

$\frac{\pi}{2} < x < \pi$

$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

$\frac{x}{2}$ is in Q1

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$

pos in Q1

$= \sqrt{\frac{1-(-5/8)}{2}} = \sqrt{\frac{1+5/8}{2}}$

$= \sqrt{\left(8/8+\frac{5}{8}\right)\frac{1}{2}} = \sqrt{\frac{13}{8}\cdot\frac{1}{2}} = \sqrt{\frac{13}{16}}$

$= \frac{\sqrt{13}}{4}$

(b) $\tan\frac{\theta}{2}$ given that $\tan\theta = \frac{\sqrt{7}}{3}$, with $180° < \theta < 260°$.

$90 < \frac{\theta}{2} < 130$

$\frac{\theta}{2}$ in Q2

$\tan\theta = \frac{\sqrt{7}}{3}$

$\sec^2\theta = \tan^2\theta + 1$

$= \left(\frac{\sqrt{7}}{3}\right)^2 + 1$

$= \frac{7}{9} + \frac{9}{9} = \frac{16}{9}$

$\sec\theta = -\frac{4}{3}$

neg because θ in Q3

$\cos\theta = \frac{-3}{4}$

$\tan\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

neg in Q2

$= -\sqrt{\frac{1-(-3/4)}{1+(-3/4)}}$

$= -\sqrt{\frac{4/4+3/4}{\frac{4}{4}-3/4}}$

$= -\sqrt{\frac{7/4}{1/4}} = -\sqrt{\frac{7}{4}\cdot\frac{4}{1}} = -\sqrt{7}$